A closure rule is *nullary* if it has the form \( x \in S \), and *unary* if it is of the form \( y \in S \implies z \in S \). Prove that if \( \Sigma \) is a collection of nullary and unary closure rules, then nonempty unions of closed sets are closed, and hence the lattice of closed sets \( C_\Sigma \) is distributive. Conclude that the subalgebra lattice of an algebra with only constants and unary operations is distributive.

Prove that the refinement relation on finite subsets of a lattice \( L \) has the following properties.

1. \( A \ll B \) implies \( \bigvee A \leq \bigvee B \).
2. The relation \( \ll \) is a quasiorde on the finite subsets of \( L \).
3. If \( A \subseteq B \) then \( A \ll B \).
4. If \( A \) is an antichain, \( A \ll B \) and \( B \ll A \), then \( A \subseteq B \).
5. If \( A \) and \( B \) are antichains with \( A \ll B \) and \( B \ll A \), then \( A = B \).
6. If \( A \ll B \) and \( B \ll A \), then \( A \) and \( B \) have the same set of maximal elements.

Draw the lattice \( \text{Co}(4) \) of convex subsets of a 4-element chain. Give the D-basis for this lattice.

Give the D-basis for five relatively small lattices.