HOMEWORK 3 - LATTICE THEORY

- A closure rule is *nullary* if it has the form $x \in S$, and *unary* if it is of the form $y \in S \implies z \in S$. Prove that if Σ is a collection of nullary and unary closure rules, then nonempty unions of closed sets are closed, and hence the lattice of closed sets $\mathcal{C}_\Sigma$ is distributive. Conclude that the subalgebra lattice of an algebra with only constants and unary operations is distributive.
- Prove that the refinement relation on finite subsets of a lattice Ł has the following properties.
  (1) $A \ll B$ implies $\bigvee A \leq \bigvee B$.
  (2) The relation $\ll$ is a quasiorder on the finite subsets of L.
  (3) If $A \subseteq B$ then $A \ll B$.
  (4) If A is an antichain, $A \ll B$ and $B \ll A$, then $A \subseteq B$.
  (5) If A and B are antichains with $A \ll B$ and $B \ll A$, then $A = B$.
  (6) If $A \ll B$ and $B \ll A$, then A and B have the same set of maximal elements.
- Draw the lattice $\mathrm{Co}(\mathbf{4})$ of convex subsets of a 4-element chain. Give the D-basis for this lattice.
- Give the D-basis for five relatively small lattices.

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