(1) (10 points) Using the limit definition of the derivative, give $f'(x)$ if $f(x) = \frac{1}{x^2+1}$.

(2) (5 points) Compute $\lim_{x \to 6} \frac{\sqrt{x-3}-2}{x-6}$. 
(3) (20 points) Take the derivative of the following, be VERY careful!

\[ f(x) = \frac{\sqrt[3]{\sin(x^2)}}{x^2 + 2x + \pi} \]

\[ k(x) = \sqrt{\cos(\sqrt{\sin(\ln(x)e^x)})} \]
(4) (10 points) Draw a function with the following properties: \( f(2) = 1, f'(2) = f'(4) = f'(-2) = 0, f''(3) = f''(0) = f''(5) = 0, f'(x) > 0 \) on \((-\infty, -2) \cup (2, 4), f'(x) < 0 \) on \((-2, 2) \cup (4, \infty), f''(x) > 0 \) on \((0, 3) \cup (5, \infty), f''(x) < 0 \) on \((-\infty, 0) \cup (3, 5)\) and \( \lim_{x \to \infty} f(x) = 0. \)
(5) (10 points) Suppose that we are selling manapuas. Suppose that if we want to sell 1000 manapuas, then we can sell them at 1 dollar each, and if we only want to sell 400 manapuas, we can sell them at 1.75 dollars. If demand is linear, give the demand equation and give how many manapuas should we make and how much do we sell them for if revenue is maximized?

(6) (10 points) We are in a hot air balloon, going straight up. Originally, there was a rope station 20ft. away from the balloon, and a rope connecting the balloon and the rope station. The rope is connected to the balloon for the entirety of our journey. We notice that we are going up at a constant rate of 5ft/s the whole time. 30 seconds after take off, how fast must we let out rope to keep the rope tight.