

MATH 241, Fall '11
Final Exam

Name:
Instructor:

INSTRUCTIONS: Write legibly. Indicate your answer clearly. Show all work; explain your answers. Answers with work not shown might be worth **zero** points. No calculators, cell phones, or cheating.

Problem	Worth	Score
1	24	
2	24	
3	10	
4	28	
5	12	
6	16	
7	16	
8	8	
9	16	
10	18	
11	16	
12	12	
Total	200	

(6) 0. **Extra Credit:** Show that for all x and y , $|\sin(x) - \sin(y)| \leq |x - y|$.

(24) 1. Compute each of the following limits or show that they do not exist. **Show your work!**

(a) $\lim_{x \rightarrow -3} \frac{x+3}{x^2+7x+12}$

(b) $\lim_{x \rightarrow 9} \frac{\sqrt{x}-3}{x-9}$

(c) $\lim_{x \rightarrow 0} \frac{x}{\sin 3x}$

(d) $\lim_{x \rightarrow 2^+} \frac{|x-2|}{2-x}$

(24) 2. Find the derivatives of each of the following functions. **Do not simplify!**

(a) $f(x) = x^2 \sqrt{\sin x}$

$$f'(x) =$$

(b) $g(x) = \frac{x^2 + x + 1}{\sqrt{x^2 + 1}}$

$$g'(x) =$$

(c) $h(x) = \int_{\sqrt{x}}^5 \sin^5 t \, dt$

$$h'(x) =$$

(10) 3. Let $f(x) = \frac{1}{3x}$. Use the *definition of the derivative* to compute $f'(2)$. **No work, no credit.**

(28) 4. Evaluate the following integrals. **Show your work!**

(a) $\int x^2 \sqrt{1 + 10x^3} \, dx$

(b) $\int_0^1 \frac{x}{\sqrt{x+1}} \, dx$

(c) $\int_0^1 (x^2 + 1)(3x - 2) \, dx$

(d) $\int_0^{\pi/2} \sin x \cos^3 x \, dx$

(12) 5. Find the equation of the tangent line to the curve $x^3 + y^3 = 9xy$ at the point $(4, 2)$. **Show your work!**

(16) 6. 100m^3 of oil is spilled when a tanker collides with a tuna boat. The resulting oil slick forms a right circular cylinder on the surface of the water. If the thickness (h) of the slick is decreasing at a rate of 0.001 m/sec, how fast is the radius (r) increasing when the slick is 0.01 m thick? Note: $V = \pi r^2 h$.

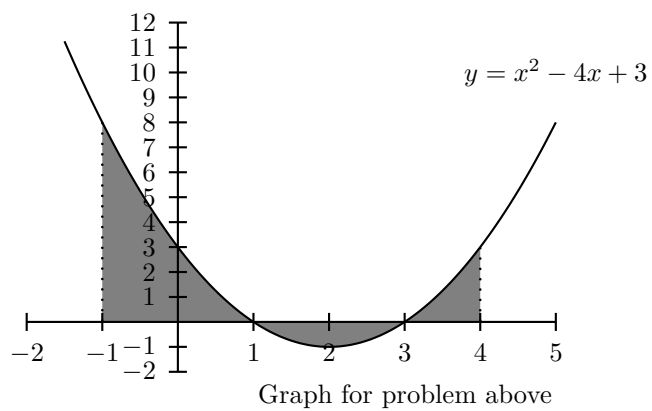
- (16) 7. A rectangular box with volume 18 ft^3 is to be built with a square base and no top. The material used for the bottom panel costs \$ 2.00 per ft^2 while the material used for the side panels cost \$ 1.50 per ft^2 . Find the minimum cost of such a box. **Justify your answer using the methods of calculus.**

- (8) 8. Set up the Riemann sum approximation to the integral $\int_0^4 x^3 dx$ by partitioning the interval $[0, 4]$ into 4 subintervals of equal length and using the right endpoint of each subinterval to calculate the height of the corresponding rectangle.

(16) 9. Let $f(x) = x^2 - 4x + 3$. The graph of $y = f(x)$ is shown below.

(a) Compute $\int_{-1}^4 f(x) dx$

(b) Find the total area between the graph and the x -axis for x between -1 and 4 .



(18) 10. Let $f(x) = \frac{x^2}{(x-3)^2}$. Answer the question below. **Show all reasoning using the methods of calculus.** Note: $f'(x) = \frac{-6x}{(x-3)^3}$ and $f''(x) = \frac{12x+18}{(x-3)^4}$.

(a) Find all points where f is not continuous.

(b) Find the intervals where f is increasing and the intervals where f is decreasing.

(c) Find the intervals where f is concave up and the intervals where f is concave down.

(d) Find all local extrema.

(e) Find all inflection points.

(f) Find the equations of all asymptotes.

(16) 11. Solve the initial value problem (In other words, find a function $y(x)$ that satisfies both equations.):

$$\frac{dy}{dx} = 3 \sin(2x) + 6 \quad \text{and} \quad y(0) = 1.$$

(12) 12. Let $f(x) = 3x^2 + 5x - 9$.

(a) Explain why f satisfies the hypotheses of the Mean Value Theorem over the interval $[0, 3]$.

(b) Find a point $c \in (0, 3)$ such that the slope of the tangent line at $(c, f(c))$ is the average rate of change of f over the interval.