

MATH241, Fall '10  
Final Exam

Name:  
Instructor:

*INSTRUCTIONS:* Write legibly. Indicate your answer clearly. Show all work; explain your answers. Answers with work not shown might be worth **zero** points. No calculators, cell phones, or cheating.

| Problem | Worth | Score |
|---------|-------|-------|
| 1       | 20    |       |
| 2       | 8     |       |
| 3       | 6     |       |
| 4       | 12    |       |
| 5       | 10    |       |
| 6       | 8     |       |
| 7       | 8     |       |
| 8       | 15    |       |
| 9       | 8     |       |
| 10      | 24    |       |
| 11      | 18    |       |
| 12      | 8     |       |
| 13      | 12    |       |
| Total   | 157   |       |

(6) 0. **Extra Credit:** Show that for all  $x$  and  $y$ ,  $|\sin(x) - \sin(y)| \leq |x - y|$ .

(20) 1. Find the derivative; do not simplify your answer.

(a)  $f(x) = \frac{\sin x}{x^3 + \cos x}$

$$f'(x) =$$

(b)  $g(x) = \tan(x^2 + \pi + \cos x)$

$$g'(x) =$$

(c)  $h(x) = \int_{x^2+1}^2 \sqrt{5+t^2} dt$

$$h'(x) =$$

(d) The equation  $x^2 + 2xy^2 - \frac{x}{y} = 6$  implicitly defines a function  $y(x)$ . Find  $\frac{dy}{dx}$  at  $(x, y) = (2, 1)$ :

$$\left. \frac{dy}{dx} \right|_{(2,1)} =$$

- (8) 2. Differentiate  $f(x) = 1/x$  at  $x = 3$  using first principles (find the limit of the appropriate difference quotient). **No credit for the use of the rules of differentiation!**

- (6) 3. Find the equation of the tangent line to the graph of the function  $f(x) = x \cos x$  at  $x = \pi/3$ .

- (12) 4. Consider a function  $y = y(t)$  that satisfies the differential equation  $y' = y(3 - y)$  and suppose that  $y(0) = 2$ .

(a) Compute  $y'(0)$  and use it to find an approximate value for  $y$  when  $t = .2$ .

(b) Compute  $y''$  and decide whether  $y$  is concave up or down at time  $t = 0$ .

- (10) 5. Evaluate the limit, or explain why it does not exist:

(a)  $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x - 1}$

(b)  $\lim_{x \rightarrow \infty} x \left( \sqrt{x^2 + 4} - \sqrt{x^2 + 1} \right)$

- (8) 6. How many real solutions does  $f(x) = 4x^3 - 6x^2 - 6x + 5$  have? Justify your answer. Do NOT try to find the solutions!

- (8) 7. A lighthouse is located on a small island 3 km away from the nearest point  $P$  on a straight shoreline and its light makes four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from  $P$ ?

(15) 8. Compute the integrals:

(a)  $\int \sec^2(3x) dx =$

(b)  $\int x^2 \sqrt{x+1} dx =$

(c)  $\int x \sin(x^2) dx =$

(8) 9. Find the area of the region bounded by the lines  $y = x$  and  $y = 8x$  and the curve  $y = 1/x^2$ .

(24) 10. Let  $f(x) = \frac{x^2 + 5}{x - 2}$

(a)  $f'(x) =$

(b) Find the critical points and intervals on which  $f$  is increasing/decreasing.

(c) Find the local extrema.

(d) Find the global maxima and minima of  $f$  on  $[3, 6]$

(e) Find all asymptotes of  $f(x)$ , horizontal, vertical, and slant.

(f) Sketch the graph of  $f(x)$ .



- (18) 11. Find the radius  $r$  and the height  $h$  of a right circular cone if its volume is maximal and its slant height is  $S = 17$ . What is the ratio  $h/r$ ?

- (8) 12. Calculate the Riemann sum for the function  $f(x) = 1/x^2$  on the interval  $[1, 5]$ . Partition the interval using the points  $x_0 = 1$ ,  $x_1 = 2$ ,  $x_2 = 3$ ,  $x_3 = 5$ . Use the midpoint in each of the subintervals as distinguished point.

(12) 13. Let  $\Omega$  be the region bounded by the line  $y = 1 - x$ ,  $x = 2$ , and  $y = 4 - x$ . Set up definite integrals representing each of the following quantities (DO NOT EVALUATE YOUR INTEGRALS!):

(a) the volume of the solid obtained by revolving  $\Omega$  around the **x-axis**.

(b) the volume of the solid obtained by revolving  $\Omega$  around the **y-axis**.

(c) the volume of the solid obtained by revolving  $\Omega$  around the line  $y = -2$ .