MATH241, Spring '11
Final Exam

Name:
Instructor:

INSTRUCTIONS: Write legibly. Indicate your answer clearly. Show all work; explain your answers. Answers with work not shown might be worth **zero** points. No calculators, cell phones, or cheating.

Problem	Worth	Score
1	20	
2	8	
3	6	
4	15	
5	10	
6	8	
7	8	
8	15	
9	8	
10	24	
11	18	
12	8	
13	12	
Total	160	

(6) 0. **Extra Credit:** Show that for all x and y, $|\sin(x) - \sin(y)| \leq |x - y|$.

(20) 1. Find the derivative; do not simplify your answer.

(a) $f(x) = \dfrac{\sqrt{x}}{x^3 + 1}$

$f'(x) =$

(b) $g(x) = \sec\left(2x + \dfrac{\pi}{4}\right)$

$g'(x) =$

(c) $h(x) = \displaystyle\int_{\sin x}^{x} \tan^2(t+1)\, dt$

$h'(x) =$

(d) $k(x) = \tan(2x)\sqrt{1 + \cos(3x)}$

$k'(x) =$

(8) 2. Differentiate $f(x) = x^3 - 3x$ at $x = 2$ using first principles (find the limit of the appropriate difference quotient). **No credit for the use of the rules of differentiation!**

(6) 3. Find the equation of the tangent line to the graph of the function $f(x) = \tan x$ at $x = \pi/3$.

(15) 4. Consider the curve C given by the equation $\sin y = \cos\sqrt{x}$.

(a) What is the slope of the curve C at the point $P = (\pi^2/4, 0)$?

(b) Let Q be another point on the curve whose x–coordinate is $\pi^2/4 + .1$. Find an approximate value for the y–coordinate of Q.

(c) Thinking of y as an implicitly defined function of x, find y'' at the point P.

(10) 5. Evaluate the limit, or explain why it does not exist:

(a) $\lim_{x\to 1} \dfrac{x-1}{x^{\frac{1}{3}}-1}$

(b) $\lim_{x\to\infty} x\left(x - \sqrt{x^2+1}\right)$

(8) 6. Give a well supported argument that every polynomial of degree three has a real root.

(8) 7. Consider a point $P = (x, y)$ that moves along the graph of the function $y = 8/x$ with a horizontal velocity of 3 units per second. At which rate does the distance between P and the origin $(0, 0)$ of the coordinate system change as the point passes through $(4, 2)$?

(15) 8. Compute the integrals:

(a) $\int x(x+2)^{15}\,dx =$

(b) $\int \tan^2(2x)\,dx =$

(c) $\int |x|\,dx =$

(8) 9. Sketch and find the area of the region that lies above the x–axis and below both parabolas, $p = 4-(x-2)^2$ and $q = 4-(x-4)^2$.

(24) 10. Let $f(x) = x(x-3)^2 = x^3 - 6x^2 + 9x$.

(a) Find the zeros of $f(x)$ and provide the intervals on which $f(x)$ is positive, respectively negative.

(b) $f'(x) =$ $f''(x) =$

(c) Find the critical points and intervals on which f is increasing/decreasing.

(d) Find the local extrema.

(e) Find the absolute maxima and minima of f on $[-1, 3.9]$

(f) Find the inflection points and the intervals on which $f(x)$ is concave, respectively down.

(g) Sketch the graph of $f(x)$.

(18) 11. Find the radius r and the height h of a right circular cylinder if its volume is maximal and it fits inside a hemisphere of radius 15. What is the ratio h/r?

(8) 12. We like to compute the Riemann sum for the function $f(x) = x^2$ on the interval $I = [1, 2]$. Specifically, partition I into four intervals of equal lengths, and use their midpoints as distiguished points in the computation.

(12) 13. Let Ω be the region bounded by the x–axis $y = 0$ and parabola $p(x) = 1 - (x-2)^2$. Set up definite integrals representing each of the following quantities (DO NOT EVALUATE YOUR INTEGRALS!):

(a) the volume of the solid obtained by revolving Ω around the **x-axis**.

(b) the volume of the solid obtained by revolving Ω around the **y-axis**. Use the shell method.

(c) the volume of the solid obtained by revolving Ω around the **y-axis**. Use the washer method.