Math 242: HW 2

Due on Thursday, June 19

Summer '14

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Problem 1

Use the Mean Value Theorem to prove that $\ln(xy) = \ln(x) + \ln(y)$ for any positive $x, y > 0$.

Problem 2

Use problem 1) and the fact proven in class, $\ln(x^n) = n\ln(x)$ for any $x > 0$, to prove that $\ln(\frac{x}{y}) = \ln(x) - \ln(y)$ for any $x, y > 0$.

Problem 3

Draw a graph of $f(t) = \frac{1}{t}$, make sure to mark the t-values 1 and 2, along with the corresponding values for $f(1)$ and $f(2)$. On **the same graph** draw a rectangle with vertices at $(1, 1/2), (2, 1/2), (2, 0), (1, 0)$. What is the area of this rectangle? Why is the area of this rectangle less than $\ln(2)$?

Problem 4

Use problem 3) to show that $\lim_{x\to\infty} \ln(x) = \infty$.

Hint: Let M be ANY positive number, if you can find an x_M such that $\ln(x_M) > M$ then (since $\ln(x)$ is increasing) you have shown that $\lim_{x\to\infty} \ln(x) = \infty$.

Problem 5

Compute the following:

a) $\displaystyle\int \frac{1}{x+1}\, dx$

b) $\displaystyle\int_0^1 \frac{x}{x^2+1}\, dx$

c) $\displaystyle\frac{d}{dx}\big(\ln(\sec(x))\big)$

d) $\displaystyle\int \frac{\sec^2(x)}{\tan(x)}\, dx$