Math 243
Spring 2019
Final
Please Study
Time Limit: 120 minutes
No Notes
No Calculators
No Funny Business

Name (Print): ____________________

Problem	Points	Score
1	20	
2	25	
3	15	
4	10	
5	10	
6	10	
7	20	
8	40	
9	20	
10	20	
11	10	
12	20	
13	20	
14	15	
15	15	
Total:	270	

1. (a) (20 points) Sketch a graph of the following:

$z = x^2 + y^2$

$x = y^2 + z^2$

$z = x^2 - y^2$

$9 = x^2 + y^2 + z^2$

$1 = \frac{z^2}{9} + x^2 + y^2$

$z^2 = x^2 + y^2$

$x^2 = z^2 + y^2$

$z^2 = x^2 + y^2 + 1$

$z^2 = x^2 + y^2 - 1$

2. Let $P = (1, 2, 0)$, $Q = (1, 1, 3)$ and $R = (0, 0, 1)$.
   (a) (5 points) Find the angle between $\overrightarrow{P}$ and $\overrightarrow{Q}$.

   (b) (5 points) Parametrize the the line segment from P to Q.

   (c) (5 points) Give an equation of the plane containing P, Q and R.

   (d) (5 points) Give the area of the triangle whose vertices are P, Q and R.

   (e) (5 points) Give an equation of a sphere whose surface contains the points P, Q and R.

3. Consider the parametric equations

$$x = \cos(2t) \quad \text{and} \quad y = 2t + \sin(2t) \quad \text{for } 0 \le t \le \pi.$$

(a) (10 points) Find the length of this curve.

(b) (5 points) Find the equation of the tangent line when $t = \frac{\pi}{4}$.

4. (a) (5 points) Find $\lim_{(x,y)\to(4,3)} \dfrac{\sqrt{x}-\sqrt{y+1}}{x-y-1}$ if it exists.

(b) (5 points) Find $\lim_{(x,y)\to(0,0)} \dfrac{x^2}{x^2+y}$ if it exists.

5. (10 points) For differentiable vector valued functions $u(t)$ and $v(t)$, prove that $\frac{d}{dt}\Big(u(t) \cdot v(t)\Big) = \frac{du}{dt} \cdot v(t) + u(t) \cdot \frac{dv}{dt}$

6. (10 points) Show that if $r(t)$ is a differentiable vector valued function and $|r(t)| = C$ for a constant C, then $r(t)$ and $\frac{dr}{dt}$ are orthogonal.

7. (20 points) Find $r(t)$ if

$$\frac{d^2r}{dt^2} = i + j - 32\ k, \quad \left.\frac{dr}{dt}\right|_{t=0} = 8\ i + 8\ j \quad \text{and} \quad r(0) = 2\ i + j + \ k$$

8. (40 points) Let
$$r(t) = \cos^3(t)\ i + \sin^3(t)\ j.$$
Find the unit tangent vector, T, the principle unit normal vector, N, the curvature, κ, the unit binormal, B, and the torsion, τ, of this curve.

9. (20 points) Let $f(x,y,z) = \dfrac{z^2 e^{zxy}}{x}$. Find f_x, f_y, f_z and f_{xyz}.

10. (20 points) a) Suppose that $r(t) = g(t)\ i + h(t)\ j$ is a vector valued function such that $f(g(t), h(t)) = c$ for some constant c. Show that ∇f and $\frac{dr}{dt}$ are orthogonal along this level curve.

11. (a) (5 points) Let $f(x, y) = x^2 - y^2 + 3$. Find on equation of the tangent plane at the point $(4, 4, 3)$.

(b) (5 points) Let $f(x, y, z) = 2x^3 + 4y^2 - z^2$. Verify that the point $(1, 1, 1)$ is on the level surface $f = 5$, then find an equation of the tangent plane at that point.

12. Let $f(x, y) = \frac{1}{\sqrt{1-x^2-y^2}}$.

(a) (5 points) Find the domain and range of $f(x, y)$.

(b) (5 points) Is the domain open/closed or neither? What is the boundary of the domain? Is the domain bounded or unbounded?

(c) (5 points) Graph the level curve $f(x, y) = 8$. Determine if $(\frac{3}{2\sqrt{2}}, \frac{3}{2\sqrt{2}})$ is on this level curve. If it is, plot ∇f on the level curve at this point.

(d) (5 points) Find find a c such that the level curve $f(x, y) = c$ contains the point $(\frac{1}{2}, \frac{1}{\sqrt{2}})$.

13. (a) (20 points) Let $f(x, y) = 8x^3 + y^3 + 6xy$. Use the second derivative test to find any local min/max or saddle points. You do not need to evaluate $f(x, y)$ at these points.

14. (15 points) Find the **cubic** (degree 3) approximation for the function

$$f(x,y) = e^{2x}\ln(1+3y)$$

centered at the origin.

15. (15 points) There will be a "Lagrange" type question similar to ones found on the homework or worksheet. Here is one I took from the book:

    Suppose that the temperature (in degrees Celsius) on the sphere $x^2 + y^2 + z^2 = 1$ is given by the function $T(x, y, z) = xyz^2$. Find the hottest and coldest points on the sphere. Would water freeze at any point on the sphere?