

## Problem 1

Recall the three-dimensional Laplace equation:

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$$

Show that the functions

$$f(x, y) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$$

and

$$g(x, y, z) = e^{3x+4y} \cos(5z)$$

are both solutions to the Laplace equation.

**Problem 2**

Let  $w = f(x, y)$  and  $x = r \cos(\theta)$  and  $y = r \sin(\theta)$ .

a) Find  $\frac{\partial w}{\partial r}$ .

b) Find  $\frac{\partial w}{\partial \theta}$ .

**Problem 3**

Let  $f(x, y) = \ln(x^2 + y^2)$  and suppose  $x = e^t$  and  $y = t^2 + t + 1$ . Find  $\frac{df}{dt}$ .

**Problem 4**

Suppose that  $w = f(u, v, x, y)$  and  $u = g(\alpha, \beta)$ ,  $v = h(\alpha, \beta)$ ,  $x = k(\alpha, \beta)$ ,  $y = l(\alpha, \beta)$ . Find  $\frac{\partial w}{\partial \alpha}$  and  $\frac{\partial w}{\partial \beta}$ .