

Math 243
Spring 2019
Practice Exam 1
Doomsday

Name (Print): _____

Time Limit: Probably Not Enough

Problem	Points	Score
1	15	
2	30	
3	40	
4	15	
5	20	
6	20	
7	20	
Total:	160	

1. (15 points) Let $P = (1, 2, 3)$ and $Q = (0, -1, 2)$.
 - a) Find the distance between P and Q .
 - b) Give the equation of a sphere, centered at P , that has the point Q on its surface.
 - c) Find the vector \overrightarrow{PQ} .
 - d) Find the angle between \overrightarrow{P} and \overrightarrow{Q} .
 - e) Parametrize (with parametric equations) the line **segment** which starts at P and ends at Q .

2. The following points define the vertices of a triangle:

$$A = (0, 0, 1) \quad B = (1, 3, -1) \quad C = (2, 2, 2)$$

- (a) (10 points) Find the area of the triangle.
- (b) (10 points) Find an equation of the plane which contains the triangle.
- (c) (10 points) Find the distance between the point $(3, 2, 1)$ and the plane from part *b*).

-
3. (a) (10 points) Let $u = i + j$, $v = i + j + k$. Find the projection of u onto $u + v$.
- (b) (10 points) The vectors $u = i + 2j$ and $v = j + 3k$ lie in a plane that goes through the point $P = (1, 0, 1)$. Give the equation of this plane.
- (c) (10 points) Give the equation of a line, perpendicular to the plane $2x + 3y + z = 6$, that goes through the point $(1, 0, 1)$.
- (d) (10 points) Find the point in space the line from part c) intersects the plane from part b).

4. (15 points) Match the surface with its equation

$$1 = x^2 + y^2 - z^2$$

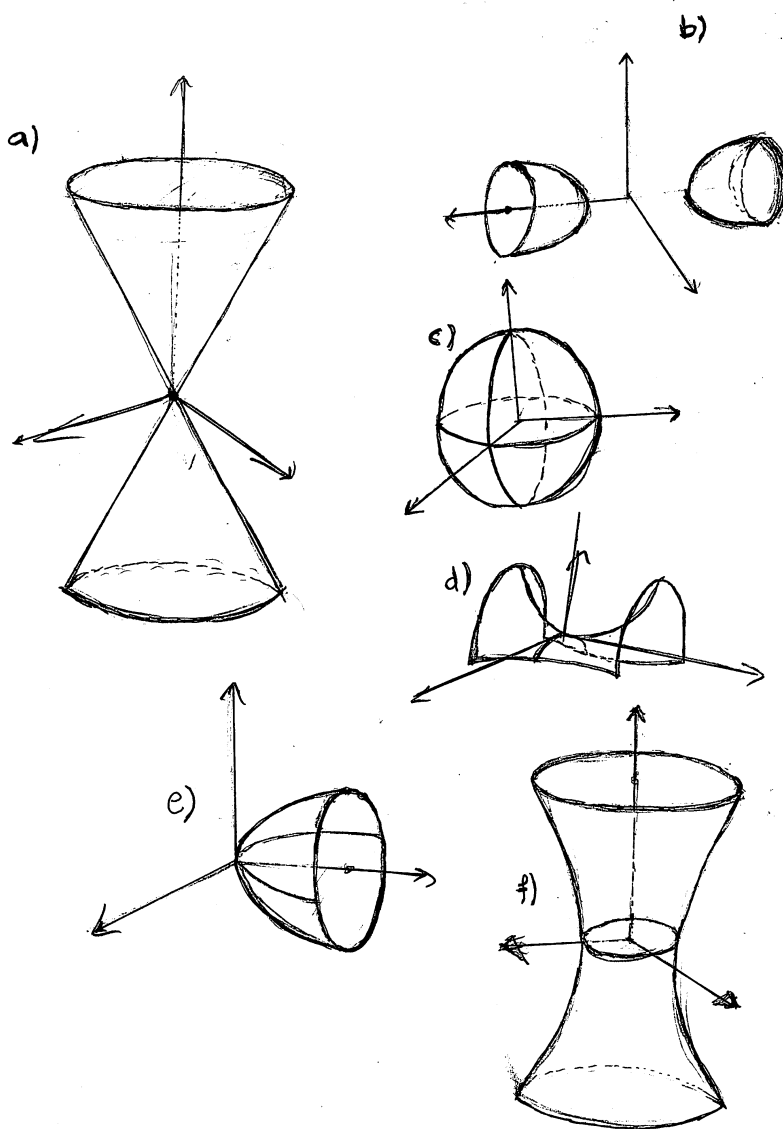
$$y = x^2 + z^2$$

$$1 = x^2 + y^2 + z^2$$

$$z = y^2 - x^2$$

$$0 = x^2 + y^2 - z^2$$

$$x^2 = z^2 + y^2 + 1$$



5. (20 points) Find the length of the curve given by the parametric equations

$$x = \cos(t) \quad \text{and} \quad y = t + \sin(t) \quad \text{for } 0 \leq t \leq \pi.$$

Also, find the equation of the tangent line when $t = \frac{\pi}{2}$.

6. (20 points) Find the length of the curve

$$x = \frac{y^3}{6} + \frac{1}{2y}$$

from $y = 2$ to $y = 3$.

7. (20 points) Consider the polar coordinate equations $r = 2(1 + \cos(\theta))$ and $r = 2(1 - \cos(\theta))$.
- a) Graph both of these curves. For both graphs, find and plot the equation of the tangent line when $\theta = \frac{\pi}{6}$.

b) Find the length of each curve.

c) Find the area between the curves in the first quadrant.