

Name:

Section: 2 4 (circle one)

1. Consider the function $f(x, y) = \sqrt{1 - x^2 - y^2}$.

a) Sketch a graph of this function and plot the point $(\frac{1}{2}, \frac{1}{2}, f(\frac{1}{2}, \frac{1}{2}))$.

b) Find ∇f . Plot the level curve of f containing the point $(\frac{1}{2}, \frac{1}{2})$ and plot the vector $(\nabla f)_{(\frac{1}{2}, \frac{1}{2})}$ at this point.

c) Find the derivative of f in the direction $u = 2i + 3j$ at the point $(0, \frac{1}{\sqrt{2}})$

d) Find the direction in which f is increasing most rapidly. Find the derivative of f in this direction.

e) Find the equation of the tangent plane to the surface at $(\frac{1}{2}, \frac{1}{2}, f(\frac{1}{2}, \frac{1}{2}))$

f) Find the equation of the normal line to the plane at this point.

2. Consider the function $f(x, y, z) = xy + xz + zy + xyz$.

a) Show that the point $(1, 1, 1)$ is on the level surface $4 = f(x, y, z)$.

b) Find the equation of the tangent plane to the level surface at this point.

c) Find the equation of the normal line at this point.

3. Consider the curve given by the intersection of the surfaces

$$14 = x^2 + y^2 + z^2 \quad \text{and} \quad 0 = x^2 - y^2 - z.$$

a) Show that $(2, 1, 3)$ is in the intersection of both the surfaces.

b) Give the equation of the tangent line to the curve of intersection at the point $(2, 1, 3)$.