Problem 1

Find the gradient field of the function $f(x, y, z) = x^2y^2 + xyz$.

Problem 2

For each given vector field \mathbf{F} , find the work done by \mathbf{F} on the curves $C_1 : \mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^4\mathbf{k}$ for $0 \le t \le 1$, and $C_2 : \mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + t\mathbf{k}$ for $0 \le t \le 1$.

$$\mathbf{F} = 2y\mathbf{i} + 2x\mathbf{j} + 4z\mathbf{k}$$

$$\mathbf{F} = \frac{1}{x^2 + 1}\mathbf{i}$$

$$\mathbf{F} = xy\mathbf{i} + yz\mathbf{j} + xz\mathbf{k}$$

Problem 3

Suppose a velocity field is given by $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$. Find the circulation and the flux around and across the ellipse $\mathbf{r}(t) = \cos(t)\mathbf{i} + 4\sin(t)\mathbf{j}$. Assume the curve is closed and only traversed once.

Problem 4

Suppose a velocity field is given by $\mathbf{F} = x\mathbf{i} - y\mathbf{j}$. Find the circulation and the flux around and across the ellipse $\mathbf{r}(t) = \cos(t)\mathbf{i} + 4\sin(t)\mathbf{j}$. Assume the curve is closed and only traversed once.