Problem 1

The polar function $r = a(1+\cos\theta)$ for $(a > 0)$ describes a cardioid. Calculating the flux across such a shape comes up in applied medical mathematical research a lot (so says my smart doctor friend). Supposedly, one vector field found in practice is given by

$$\mathbf{F} = \left(3xy - \frac{x}{1+y^2}\right)\mathbf{i} + \left(e^x + \tan^{-1}(y)\right)\mathbf{j}.$$

Pick some points in the xy-plane (at least 6) and plug them into this field to make an educated hypothesis about the flux of this field across said cardioid (make sure to include your sketch). Then check your hypothesis by computing the flux using green's theorem. For extra credit, compute the circulation.

Problem 2

Let $\mathbf{F} = \left(y + e^x \ln(y)\right)\mathbf{i} + \dfrac{e^x}{y}\mathbf{j}$ and C be the curve best described as the boundary of the region bounded above by $y = 3 - x^2$ and below by $y = x^4 + 1$. Find the circulation of $\mathbf{F}$ around C.

Problem 3

Use Green's Theorem to evaluate the following:

$\oint_C (y^2 \ dx + x^2 \ dy)$, where C is the triangle bounded by $x = 0$, $x + y = 1$, and $y = 0$.

$\oint_C (3y \ dx + 2x \ dy)$, where C is the boundary of $0 \leq x \leq \pi$ and $0 \leq y \leq \sin(x)$

$\oint_C (6y + x) \ dx + (y + 2x) \ dy$, where C is the circle $(x - 2)^2 + (y - 3)^2 = 4$.