Problem 1

Let $\mathbf{F} = x^2\mathbf{i} + 2x\mathbf{j} + z^2\mathbf{k}$ and C be the ellipse $4x^2 + y^2 = 4$ in the xy-plane traversed counterclockwise when viewed from above. Use a surface integral in Stokes' Theorem to find the circulation of $\mathbf{F}$ around C .

Problem 2

Let $\mathbf{F} = x^2y^3\mathbf{i}+\mathbf{j}+z\mathbf{k}$ and C the intersection of the cylinder $x^2+y^2 = 4$ and the hemisphere $x^2+y^2+z^2 = 16$, $z \geq 0$, counterclockwise when viewed from above. Use a surface integral in Stokes' Theorem to find the circulation of $\mathbf{F}$ around C.

Problem 3

Let $\mathbf{n}$ be the outer normal unit vector (away from the origin) of the parabolic shell

$$S: \quad 4x^2 + y + z^2 = 4, \quad y \geq 0,$$

and let

$$\mathbf{F} = \left(-z + \frac{1}{2+x} \right)\mathbf{i} + \left(\tan^{-1}(y) \right)\mathbf{j} + \left(x + \frac{1}{4+z} \right)\mathbf{k}.$$

Find the value of

$$\iint_S (\nabla \times \mathbf{F}) \bullet \mathbf{n} \, d\sigma$$

.

Problem 4

Let $\mathbf{F} = 2z\mathbf{i} + 3x\mathbf{j} + 5y\mathbf{k}$, and S the surface parametrized by $\mathbf{r}(r,\theta) = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j} + (4 - r^2)\mathbf{k}$ for $0 \leq \theta \leq 2\pi$, $0 \leq r \leq 2$. Calculate the flux of $\nabla \times \mathrm{F}$ across S in the direction of the outward normal $\mathbf{n}$. Hint: Use Stokes' Theorem.