Problem 1

Let $\mathbf{F} = x^2 \mathbf{i} + 2xz \mathbf{j} + z^2 \mathbf{k}$ and $C$ be the ellipse $4x^2 + y^2 = 4$ in the $xy$-plane traversed counterclockwise when viewed from above. Use a surface integral in Stokes’ Theorem to find the circulation of $\mathbf{F}$ around $C$.

Problem 2

Let $\mathbf{F} = x^2y^3 \mathbf{i} + \mathbf{j} + z \mathbf{k}$ and $C$ the intersection of the cylinder $x^2 + y^2 = 4$ and the hemisphere $x^2 + y^2 + z^2 = 16$, $z \geq 0$, counterclockwise when viewed from above. Use a surface integral in Stokes’ Theorem to find the circulation of $\mathbf{F}$ around $C$. 
Problem 3

Let \( \mathbf{n} \) be the outer normal unit vector (away from the origin) of the parabolic shell

\[
S : \quad 4x^2 + y + z^2 = 4, \quad y \geq 0,
\]

and let

\[
\mathbf{F} = \left( -z + \frac{1}{2 + x} \right) \mathbf{i} + \left( \tan^{-1}(y) \right) \mathbf{j} + \left( x + \frac{1}{4 + z} \right) \mathbf{k}.
\]

Find the value of

\[
\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, d\sigma.
\]

Problem 4

Let \( \mathbf{F} = 2z \mathbf{i} + 3x \mathbf{j} + 5y \mathbf{k} \), and \( S \) the surface parametrized by \( \mathbf{r}(r, \theta) = r \cos \theta \mathbf{i} + r \sin \theta \mathbf{j} + (4 - r^2) \mathbf{k} \) for \( 0 \leq \theta \leq 2\pi, \quad 0 \leq r \leq 2 \). Calculate the flux of \( \nabla \times \mathbf{F} \) across \( S \) in the direction of the outward normal \( \mathbf{n} \). Hint: Use Stokes’ Theorem.