251 - Worksheet 1

Name: Curlee Solutions

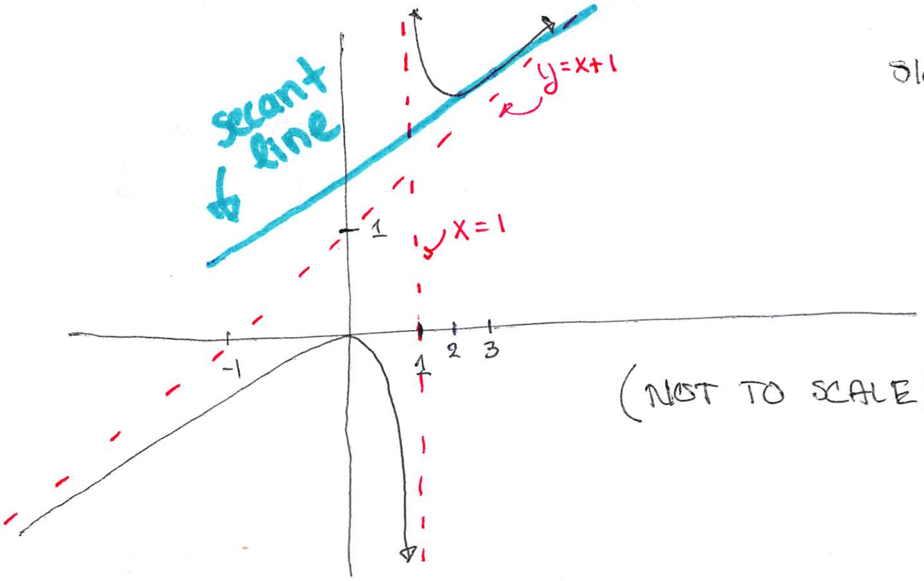
Sketch a reasonable graph of $f(x) = \dfrac{x^2}{x-1}$ and label the asymptotes. Determine the slope of the secant line between the points $(2, f(2))$ and $(3, f(3))$ and plot this line on your graph.

Since

$$\begin{array}{r} x+1 \\ x-1 \overline{\big) \; x^2 } \\ \underline{-(x^2 - x)} \\ x \\ \underline{-(x-1)} \\ +1 \end{array}$$

, $\dfrac{x^2}{x-1} = x + 1 + \dfrac{1}{x-1}$ and therefore

our function has a slant asymptote of $y = x+1$. At $x=1$ the denom. $= 0$ and numerator $\neq 0$. $\therefore$ $x = 1$ is a vertical asymptote.

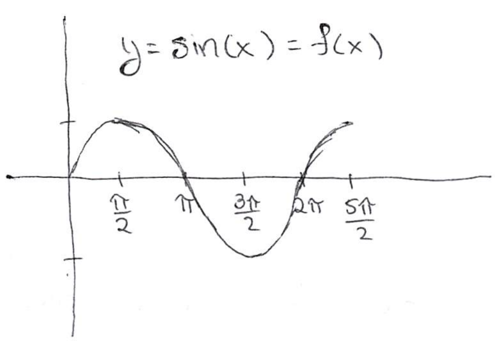


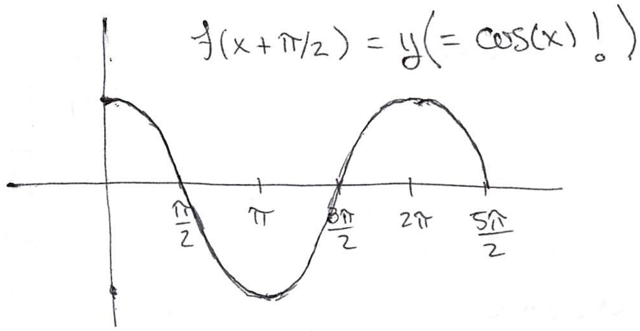
slope of the secant line:

$$\frac{f(3) - f(2)}{3-2} = \frac{1}{2}$$

(NOT TO SCALE!)

Sketch a graph of $f(x) = \sin(x)$ on the interval $[0, \frac{5\pi}{2}]$, then sketch a graph of $f(x + \pi/2)$ on the interval $[0, \frac{5\pi}{2}]$ (does this graph look familiar?!)





Let $f(x) = \frac{1}{x+1}$. Simplify $\dfrac{f(x+h) - f(x)}{h}$ to an expression that does not have a problem with $h = 0$.

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} = \frac{\left(\frac{x+1 - (x+h+1)}{(x+h+1)(x+1)}\right)}{h}$$

$$= \frac{-h}{h(x+h+1)(x+1)}$$

$$= \frac{-1}{(x+h+1)(x+1)}$$

This does not have a problem w/ $h = 1$!

Let $f(x) = ax^2 + bx + c$. Determine a condition on a, b and c that guarantees real roots of $f(x)$, then give a magic formula for the roots. (show all your work)

If $ax^2 + bx + c = 0$, then $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ and thus,

$$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$$\Leftrightarrow \quad x + \frac{b}{2a} = \pm\sqrt{\frac{b^2}{4a^2} - \frac{c}{a}}$$

$$\Leftrightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Therefore, if $b^2 - 4ac \geq 0$ then $f(x)$ has real roots, which, are given by the magic formula above.