Math 307
Spring 2019
Exam 1 - Practice
Due: 2/20/19
Time Limit: 50 Minutes

Name (Print): ____________________

Problem	Points	Score
1	20	
2	10	
3	10	
4	10	
5	40	
6	25	
7	20	
8	40	
9	20	
10	20	
11	20	
12	20	
13	20	
14	20	
15	10	
16	20	
Total:	325	

1. (20 points) Which of the following matrices are in reduced row-echelon form?

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}, \begin{bmatrix}1&0&0&8\\0&1&0&5\\0&0&1&1\end{bmatrix}, \begin{bmatrix}1&0&0&1\\0&0&0&0\\0&0&1&0\end{bmatrix}, \begin{bmatrix}1&0&0&2\\0&0&1&0\\0&0&0&0\end{bmatrix}, \begin{bmatrix}1&2&3&4&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}, \begin{bmatrix}1&0&0&0\\0&1&1&0\\0&0&0&1\end{bmatrix},$$

$$\begin{bmatrix}1&0&0&1\\0&1&1&0\\0&0&0&1\end{bmatrix}.$$

2. (10 points) Prove that $B^T B$ is always a symmetric matrix.

3. (10 points) Let $A = \begin{bmatrix}2&0&0\\0&1&0\end{bmatrix}$ and $B = \begin{bmatrix}2&0\\0&1\\2&3\end{bmatrix}$. Find $(AB)^T$.

4. (10 points) Let $A = \begin{bmatrix}1&2&0\\0&-1&1\\0&0&1\end{bmatrix}$, find A^{-1}.

5. (a) (5 points) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$

(b) (5 points) $\begin{bmatrix} 2 & 4 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} =$

(c) (5 points) If A is a 4×3 and B is a 3×4 matrix, what are the dimensions of AB and BA?

(d) (5 points) True of false: $\det(AB) = \det(A)\det(B)$

(e) (5 points) True of false: $\det(A + B) = \det(A) + \det(B)$

(f) (5 points) Prove or provide a counter example to the following statement: If A is invertible and B is invertible then $A + B$ is invertible.

(g) (5 points) Prove or provide a counter example to the following statement: If A is invertible and B is invertible then AB is invertible.

(h) (5 points) For the vectors $v_1, ..., v_n$, what is $\text{span}(v_1, ..., v_n)$?

6. (a) (15 points) Prove that A and B are row equivalent if and only if there is an invertible matrix C such that $CA = B$.

(b) (10 points) Compute

$$\det\left(\begin{bmatrix} 1 & 0 & 2 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 3 & 4 & 1 \\ -1 & -3 & -4 & -1 \end{bmatrix}\right)$$

7. (a) (10 points) Compute

$$\det\left(\begin{bmatrix} -1 & 0 & 0 & 2 \\ 1 & 1 & 2 & 1 \\ 1 & 0 & 0 & 1 \\ 2 & 0 & 4 & 0 \end{bmatrix}\right)$$

(b) (10 points) Suppose A is a 5×5 matrix such that $\det(A) = 2$. Let $E = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$.

What is $\det(EA) =$?

8. (a) (10 points) Show that $x^2 + 1, x - 1$ are linearly independent.

(b) (10 points) Show that the vectors $x^2 + 1, x - 1$ do not span P_2.

(c) (10 points) Show that $x^2 + x + 1,\ x^2 - 1,\ x + 5,\ 4$ are linearly **dependent**.

(d) (10 points) Suppose that we have $v_1, ..., v_n \in \mathbb{R}^n$. Let A be the matrix whose i-th column is the vector v_i (recall that the notation for this is $A = [v_1...v_n]$). If $\det(A) \neq 0$, explain why $v_1, ..., v_n$ form a basis for $\mathbb{R}^n$.

9. (20 points) Suppose that

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Determine if A is invertible by computing $\det(A)$. If A *is* invertible, express A^{-1} as a product of elementary matrices.

10. (20 points) State 5 properties that are all equivalent to a matrix being invertible.

11. (20 points) Let

$$A = \begin{bmatrix} 1 & 0 & -1 & 0 & 1 & 0 & 3 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

Find a basis for the null space and row space of A, the dimension of the null space of A, and the rank of A. Explain why the standard basis for $\mathbb{R}^4$ is a basis for $CS(A)$.

12. (20 points) Prove that if A is an $n \times n$ matrix and $\text{Rank}(A) = n$, then A is invertible.

13. (20 points) For a matrix A, prove that the set of vectors $\{X \in \mathbb{R}^n : AX = 0\}$ form a vector space.

14. (20 points) Prove that if A and B are row equivalent, then $NS(A) = NS(B)$.

EXTRA CREDIT:

(10 points) Define non-standard operations on $\mathbb{R}$ that satisfy all the properties of a vector space EXCEPT property 7.

(20 points) Define non-standard operations on $\mathbb{R}$ that satisfy all the properties of a vector space EXCEPT property 8.