Problem 1

Define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by

$$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x + y \\ x - 4y \end{bmatrix}$$

1. Determine if T is a linear transformation.

2. Find a matrix A such that $T(X) = AX$ for all $X \in \mathbb{R}^2$.

Problem 2

Define $T : \mathbb{R}^3 \to \mathbb{R}^3$ by

$$T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x - y + z \\ x + z \\ y - 3z \end{bmatrix}$$

1. Determine if T is a linear transformation.

2. Find a matrix A such that $T(X) = AX$ for all $X \in \mathbb{R}^3$.

Problem 3

Define $T : P_2 \to P_1$ by

$$T(ax^2 + bx + c) = 2ax + b.$$

Determine if T is a linear transformation, then find $\ker(T)$ and range(T).

Problem 4

Define $T : \mathbb{R}^3 \to \mathbb{R}^2$ by

$$T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x + y + 5z \\ x + 2y + 8z \end{bmatrix}$$

1. Determine if T is a linear transformation.

2. Find a matrix A such that $T(X) = AX$ for all $X \in \mathbb{R}^3$.

3. Find a basis for $\ker(T)$.

4. Find a basis for range(T).