# Problem 1

Suppose that $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation and

$$T\begin{bmatrix}1\\1\\0\end{bmatrix} = \begin{bmatrix}1\\2\\1\end{bmatrix}, \quad T\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}1\\0\\2\end{bmatrix}, \quad T\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}2\\2\\3\end{bmatrix}.$$

1. Determine $T\begin{bmatrix}1\\2\\3\end{bmatrix}$. (Note: you can do part 2 first and deduce this from it, if you want.)

2. Determine $T\begin{bmatrix}x\\y\\z\end{bmatrix}$

(Problem 1 Continued)

3. Find a matrix $A$ such that $T(X) = AX$ for all $X \in \mathbb{R}^3$.

4. Find a basis for $\ker(T)$.

5. Find a basis for $\text{range}(T)$.