Problem 1

Suppose that A is a matrix whose characteristic polynomial is $(\lambda - 2)^2(\lambda + 1)^2$, find all possible Jordan Normal Forms of A (up to permutation of the Jordan blocks).

Problem 2

Suppose that A is a matrix whose characteristic polynomial is $(\lambda - 4)^4(\lambda - 1)^2$, find all possible Jordan Normal Forms of A (up to permutation of the Jordan blocks).

Problem 3

Suppose that A is a matrix whose characteristic polynomial is $(\lambda - 2)^2(\lambda + 1)^2$, and $\dim(E_2) = 1$ and $\dim(E_{-1}) = 2$. Find the Jordan Normal Form of A.

Problem 4

Suppose that A is a matrix whose characteristic polynomial is $(\lambda - 3)^2(\lambda - 5)$, and A is not diagonalizable. Find the Jordan Normal Form of A.

Problem 5

Suppose that A is an $n \times n$ square matrix with only one eigenvalue, r. Prove that if $A = rI$, then A is diagonalizable.

Problem 6

Suppose that A is an $n \times n$ square matrix with only one eigenvalue, r. Prove that if A is diagonalizable, then $A = rI$. Hint: (here is one way to prove this fact, there are many other ways) If A is diagonalizable, then $\dim(E_r) = n$. Notice that E_r consists of vectors in $\mathbb{R}^n$ and therefore each standard basis vector in $\mathbb{R}^n$ is an eigenvector with eigenvalue r. Now notice that $A = AI = A[e_1 e_2 \cdots e_n]$ and use block multiplication to conclude the result.