Problem 1

On the set of vectors $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \in \mathbb{R}^2$, define an addition by

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \oplus \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 + x_1 x_2 \\ y_1 + y_2 + y_1 y_2 \end{bmatrix}$$

and a scalar multiplication by

$$k \odot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} kx \\ ky \end{bmatrix}.$$

Determine if this is a vector space.

Problem 2

On the set of vectors $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \in \mathbb{R}^2$, with x_1 in $\mathbb{R}$, and y_1 in $\mathbb{R}^+$ (meaning $y_1 > 0$) define an addition by

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \oplus \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ y_1 y_2 \end{bmatrix}$$

and a scalar multiplication by

$$k \odot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} kx \\ y^k \end{bmatrix}.$$

Determine if this is a vector space. If it is, make sure to explicitly state what the 0 vector is.