Problem 1

Verify that $\alpha = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$. Then, for $v = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$, find $[v]_\alpha$.

Problem 2

Explain why $\alpha = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$ does not form a basis for $\mathbb{R}^3$. Check that the two vectors are linearly independent, and find another vector, v, such that $\beta = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v \right\}$ *is* a basis for $\mathbb{R}^3$.

Problem 3

For the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix}$, find a basis for $NS(A)$ and $\dim(NS(A))$.

Problem 4

For the matrix $A = \begin{bmatrix} 1 & 0 & 0 & 4 & 5 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, find a basis for $NS(A)$ and $\dim(NS(A))$.