Problem 1

Determine if $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & -2 \\ 0 & 0 & 2 \end{bmatrix}$ is diagonalizable. If it is, find a diagonal matrix D, and an invertible matrix P such that $D = P^{-1}AP$.

Problem 2

Determine if $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is diagonalizable. If it is, find a diagonal matrix D, and an invertible matrix P such that $D = P^{-1}AP$.

Problem 3

Determine if $A = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$ is diagonalizable. If it is, find a diagonal matrix D, and an invertible matrix P such that $D = P^{-1}AP$.

Problem 4

Suppose that A is a matrix whose characteristic polynomial is $(\lambda - 2)^2(\lambda + 1)^2$, find all possible Jordan Normal Forms of A (up to permutation of the Jordan blocks).

Problem 5

Suppose that A is a matrix whose characteristic polynomial is $(\lambda - 4)^4(\lambda - 1)^2$, find all possible Jordan Normal Forms of A (up to permutation of the Jordan blocks).

Problem 6

Suppose that A is a matrix whose characteristic polynomial is $(\lambda - 2)^2(\lambda + 1)^2$, and $\dim(E_2) = 1$ and $\dim(E_{-1}) = 2$. Find the Jordan Normal Form of A.

Problem 7

Suppose that A is a matrix whose characteristic polynomial is $(\lambda - 3)^2(\lambda - 5)$, and A is not diagonalizable. Find the Jordan Normal Form of A.

Problem 8

Suppose that A is an $n \times n$ square matrix with only one eigenvalue, r. Prove that if $A = rI$, then A is diagonalizable.

Problem 9

Suppose that A is an $n \times n$ square matrix with only one eigenvalue, r. Prove that if A is diagonalizable, then $A = rI$. Hint: If A is diagonalizable, then $\dim(E_r) = n$. Notice that E_r consists of vectors in R^n and therefore each standard basis vector in R^n is an eigenvector with eigenvalue r. Now notice that $AI = A[e_1 e_2 \cdots e_n]$ and use block multiplication to conclude the result.