Math 307
Spring 2019
Exam 3 - Practice
4/17/19
Time Limit: 50 min.

Name (Print): _____

Problem	Points	Score
1	10	
2	15	
3	55	
4	35	
5	40	
Total:	155	

1. Consider the system of differential equations:

$$y_1' = 2y_1$$
$$y_2' = -y_2$$
$$y_3' = 15y_3$$

(a) (5 points) Find the general solution, Y_H.

(b) (5 points) Solve the initial value problem $Y(1) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

2. (15 points) Suppose that Z is a solution to $Y' = BY$, and that A and B are similar matrices. Prove that there exists an invertible matrix P such that PZ is a solution to $Y' = AY$.

3. Consider the system of differential equations:

$$y_1' = y_1 + 2y_2 + y_3$$
$$y_2' = 2y_1 + y_2 + y_3$$
$$y_3' = y_3$$

(a) (5 points) Write the system in the form $Y' = AY$.

(b) (10 points) For the matrix A, find the eigenvalues.

(c) (10 points) For each eigenvalue, find a corresponding eigenvector. In other words, find the eigenpairs.

(d) (5 points) Find an invertible matrix P, and diagonal matrix D such that $P^{-1}AP = D$.

(e) (5 points) Find the general solution to the system $Y' = DY$.

(f) (10 points) Find the general solution to the system $Y' = AY$.

(g) (10 points) Solve the initial value problem $Y(0) = \begin{bmatrix} -4 \\ 0 \\ 3 \end{bmatrix}$ for $Y' = AY$.

4. (35 points) Let $A = \begin{bmatrix} -2 & -4 \\ 5 & 2 \end{bmatrix}$. Give a real-valued general solution to the equation $Y' = AY$.

5. Suppose that the velocity of an object is given by the vector

$$v = \begin{bmatrix} 3x + 2y + z \\ 2y + 3z \\ 2z \end{bmatrix}$$

where x, y and z are the coordinates of the object's position (they are functions of time).

(a) (30 points) Find a general solution for the object's position. (part b) is on the next page)

(b) (10 points) Give the object's position when $t = 1$ if it's position is $(-7, 2, 3)$ when $t = 0$.