Math 241: HW 3

Due on Monday, September 9

Fall '13

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Problem 1

Using the definition of the limit, **prove** that

$$\lim_{x\to 1} 5x + 7 = 12.$$

Problem 2

In this problem we will use the sandwich theorem to prove that

$$\lim_{x\to 0} |x| \sin\left(\frac{1}{x}\right) = 0.$$

a)(investigation) Draw a graph of $f(x) = |x|\sin(1/x)$ (you may use a computer). Explain why we can't "plug in" the number 0 but deduce (from the graph) that this limit exists at 0 and also deduce that it is a reasonable hypothesis that the limit as $x \to 0$ exists and is indeed 0. (what a terrible sentence)

b)(further investigation) Draw a graph of $g(x) = \sin(1/x)$ and deduce that we may NOT use the product rule for limits (otherwise this problem would be easy). (hint: to use this rule, the BOTH limits must exist)

c)(the proof) Remember that $-1 \le \sin(\alpha) \le 1$ for any $\alpha \in \mathbb{R}$ (call this inequality $(*)$). Multiply $(*)$ by $|x|$ to obtain another (extremely valuable) inequality. Invoke the sandwich theorem.

Problem 3

Come up with another example of a function which **requires** the sandwich theorem to compute, then use the sandwich theorem to compute it and explain why it is required (meaning, explain why we cant just use our regular limit laws). Note: This is more or less a creative endeavor however, there is an example that is definitely the "holy grail" of the sandwich theorem. Any student who finds the "holy grail" and can explain why it is the "holy grail" will receive 5 extra points on the midterm (out of 100) and of course, my eternal congratulations.... or possibly just a lollipop...