Quiz 4

NAME: SCORE:

Problem 1.(20 points.) We are standing in the dessert, and in dire need of water. We are **standing on a road** running east to west, this is the only road everything else around us is flat sand. There is **water exactly 2 miles east and 2 miles north from where we stand**. We can **run on the road at 10 mph** and we can **run across the sand at 6 mph**. It is clear (and you don't need to show this) that the fastest way to water is running along the road at first and then at some point turning and running in a straight line towards the water. How long do we run on the road to minimize the total time it takes us to find water?

Problem 2.(20 points.) You are managing a mental institution and You find that releasing bunnies throughout your institution lowers the monthly average for mental breakdowns. This works for awhile, and you keep releasing more and more bunnies in the hallways and common areas, you are hailed as the best mental institution manager, **ever**. Eventually, you realize that after a certain number of bunnies... your bunny therapy is, unfortunately, counteractive. The bunnies start to eat everything, poo is everywhere, it smells, and then mental breakdowns start to increase. You even had one patient tell you **"...at some point doctor it's mo' bunny, mo' problems..."**.. You then find that the equation for the number of breakdowns per month, $g(x)$, given the number of bunnies, x, you have released is

$$g(x) = 9.21 + \int_2^x -\frac{1}{t} + \frac{1}{10,000} \quad dt$$

Minimize this function to find the optimal amount of bunnies and **use a derivative test to justify** that this number of bunnies really does **minimize** the number of breakdowns. Given that

$$\int_2^{10,000} -\frac{1}{t} + \frac{1}{10,000} \quad dt \approx -8.21$$

how many breakdowns should we have per month with this optimal number of bunnies?

Problem 3.(20 points.) It's dark outside. A $7ft.$ tall (drunk) man walks toward a $21ft.$ tall street light (with a light on the top) at a rate of $3ft./sec.$ (this is slow, he's drunk). The man has a shadow that is made by the lamp, there are no other sources of light around. The shadow shrinks as the man gets closer to the street light. How fast is the shadow shrinking when the man is $14ft$ away from the street light?

Problem 4.(20 points.) Compute the following:

$$\int_{1/16}^{1/9} \frac{\sec^2(\pi\sqrt{x})}{\sqrt{x}} dx$$

$$\frac{d}{dx} \int_{x^3}^{2} \sqrt{1-t^2}\,dt$$