Math 241 Final Exam

Fall, 2012

Name: ___________________________   ID: ___________________________

Signature: ______________________   Instructor: ______________________

Directions: Part A has 10 problems worth two points each. No partial credit will be awarded. Part B has 10 problems, with partial credit. Give complete and careful solutions. Show all of your work on the exam and provide as much detail as you can.

Part A           Part B
Page A1 ___________          Page B1 ___________
Page A2 ___________          Page B2 ___________
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TOTAL _______________          Page B10 ___________

GRAND TOTAL ___________
PART A - This section contains 10 problems worth two points each. No partial credit will be awarded.

1. Sketch the graph of an increasing function that is defined and continuous on \((-\infty, \infty)\), and differentiable everywhere except at \(x = 0\) and at \(x = 2\).

2. Find the limit \(\lim_{x\to 3} \frac{x^2 + 2x - 15}{x^2 - 2x - 3}\).

3. Find the limit \(\lim_{x\to 1^-} \frac{|x - 1|}{x - 1}\).
4. Precisely state the “Intermediate Value Theorem for Continuous Functions”.

5. Find the limit \( \lim_{x \to -\infty} \frac{\sqrt{4x^2 + x}}{x + 2} \).

6. Find the limit \( \lim_{x \to 0} \frac{\sin(7x)}{4x} \).

7. What is the absolute minimum value of the function \( f(x) = x^4 - 2x^2 \) on the interval \([-2, 2]\)?
8. Find the general antiderivative of $\cos(2x) + x^2$.

9. Use four rectangles of equal width and the right-endpoint values to find the Riemann sum for $f(x) = x^3 + x$ on the interval $[0, 4]$.

10. Evaluate: $\int_{1}^{8} \frac{1}{\sqrt{x}} \, dx$
PART B - This section contains 10 problems, with partial credit. Give complete and careful solutions. Show all of your work on the exam and provide as much detail as you can.

1. (8 pts.) Using only the definition of “derivative”, find $f'(x)$, where $f(x) = \sqrt{2x}$.

Answer _____________________
2. (6 pts.) Find the equation of the line tangent to the graph of

\[ y^4 - 3xy^3 + y^2 = 6x^3 - 5x^2y \]

at the point \((1, 2)\).
3. (8 pts.) Water is withdrawn from a conical reservoir 20 feet in diameter and 20 feet deep (vertex down) at a constant rate of 8 cubic feet per minute. How fast is the water level falling at the instant when the depth of the water is 8 feet? [Hint: $V_{\text{cone}} = \frac{1}{3} \pi r^2 h$].

Answer ______________________
4. (8 pts.) Use differential approximation, or the linearization method, to estimate $\sqrt{15.5}$. 

Answer ______________________
5. (10 pts.) Given $f(x) = \frac{1 - x^2}{1 + x^2}$ with $f'(x) = \frac{-4x}{(1 + x^2)^2}$ and $f''(x) = \frac{12x^2 - 4}{(1 + x^2)^3}$

1 pt. ___________ List any horizontal or vertical asymptotes of $f(x)$.

1 pt. ___________ On what interval(s) is $f(x)$ increasing?

1 pt. ___________ On what interval(s) is $f(x)$ concave down?

1 pt. ___________ Identify any local extrema.

1 pt. ___________ Locate the $x$-coordinate of any points of inflection.

5 pts. Sketch the graph of $f(x)$, marking clearly the asymptotes, coordinates of any local extrema and points of inflection.
6. (8 pts.) A fiber board shipping crate with square base and top is constructed with double thickness on the bottom for added strength. If the volume of the crate is 96 cubic feet, find the dimensions which will minimize the needed material.

Answer ____________________________
7. (8 pts.) If \( f(x) = \int_{1}^{x} \sqrt{1 + \sin t} \, dt \), what is \( f''(0) \)?

Answer ______________________
8. (8 pts.) Evaluate the following integral: \[ \int_{\pi/3}^{\pi/3} \sin x (4 + 3 \cos x) \, dx. \]

Answer ____________________
9. (8 pts.) Find the area of the region bounded by the parabolas, $y = x^2 - 1$ and $y = 1 - x^2$.

Answer ___________________________
10. (8 pts.) Consider the region bounded by $y = x^2 + 1$, and $y = 2$. Find the volume formed when this region is rotated about the $x$-axis.

Answer _______________________