

What is required of a function in order for it to have an inverse? Why do we require this?

Suppose that  $f$  is a differentiable, one-to-one function and  $f(a) = b$ . Express  $(f^{-1})'(b)$  in terms of  $f'(a)$ .

Give the domain and range of  $f(x) = \tan^{-1}(x)$ . What is  $f'(1)$ ?

State the definition of  $\ln(x)$  using a definite integral. Explain how we defined the number  $e$  this semester.

Evaluate  $\frac{d}{dx} \int_1^x \frac{1}{t} dt$ .

Integrate:  
 $\int_1^9 \frac{4}{x-10} dx$

$$\int \frac{x^2}{x^3+71} dx$$

$$\int_1^{e^2} \frac{1}{\ln(x)x} dx$$

$$\int \tan(x) dx$$

$$\int_0^1 \frac{e^x}{e^x+1} dx$$

$$\int x e^{x^2} dx$$

**Take the derivative of the following.**

$$f(x) = (x^2 + 1)^x$$

$$g(x) = 4^x \ln(x)$$

$$h(x) = \ln(\sin(e^x))$$

$$k(x) = \sin^{-1}(2^x)$$

$$l(x) = (\sin(x))^x$$

$$m(x) = \arctan(e^{2x})$$

$$n(x) = x^{\sqrt{3}}(\sqrt{3})^x$$

(this one is slightly tricky, it is included for fun)  $h(x) = x^{x^x}$

Use the Laws of Logarithms to simplify the following as much as possible (hint: beware of trickery) :

$$\ln(x^2) \ln(e^5)$$

$$\frac{\ln(64)}{\ln(2)}$$

$$25^{\log_5(6)}$$

$$\ln[x^4(\sin(x^2)) - x^3e^x]$$

$$\ln\left(\frac{x^4(\sin(x^2))}{\pi^3e^x}\right)$$

$$\text{Simplify: } \sin(\tan^{-1}(8))$$

$$\int \frac{1}{1+5x^2} dx$$

$$\int \frac{1}{\sqrt{1-5x^2}} dx$$

Compute the following limits, and show your work. (Even if you know the answer immediately, show all the required work, e.g. using L'Hôpital's Rule.)

$$\lim_{n \rightarrow \infty} \frac{\ln(n)}{n}$$

$$\lim_{n \rightarrow \infty} \left(1 - \frac{4}{n}\right)^n$$

$$\lim_{x \rightarrow 0^+} x \ln\left(\frac{10}{x}\right)$$

$$\lim_{x \rightarrow \frac{\pi}{2}^+} \frac{1}{\cos(x)} - \frac{1}{x - \frac{\pi}{2}}$$

$$\lim_{n \rightarrow \infty} e^n \ln\left(1 - \frac{1}{n}\right)$$

$$\lim_{n \rightarrow \infty} \frac{3 + (.2)^n}{7 - \frac{1}{n}}$$

$$\lim_{n \rightarrow \infty} \frac{10^n}{n}$$

**Integrate.**

$$\int x \sin(x) \, dx$$

$$\int \tan^{-1}(x) \, dx$$

$$\int x e^x \, dx$$

$$\int \sin(2x) e^x \, dx$$

$$\int x^2 \ln(x) \, dx$$

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$$\int \cos^5(x) \sin^3(x) dx$$

$$\int \cos^4(x) \sin^3(x) dx$$

$$\int \cos^2(x) \sin^2(x) dx$$

$$\int \sec^4(x) dx$$

$$\int \tan^3(x) dx$$

$$\int \frac{\sqrt{x^2 - 4}}{x} \, dx$$

$$\int \frac{1}{\sqrt{x^2 + 9}} \, dx$$

$$\int_0^{1/2} \frac{4x^2}{(1 - x^2)^{3/2}} \, dx$$

$$\int \frac{e^x}{(1 + e^{2x})^{3/2}} \, dx$$

$$\int \frac{5x - 13}{(x - 3)(x - 2)} dx$$

$$\int \frac{x + 4}{x^2 + 2x + 1} dx$$

$$\int \frac{1}{1 - 4x^2} dx$$

$$\int \frac{x^3}{x^2 + 2x} dx$$

$$\int \frac{1}{x^3 + x} dx$$

$$\int_0^1 \frac{1}{x-1} \, dx$$

$$\int_1^\infty \frac{1}{x^2} \, dx$$

$$\int_0^1 \ln(x) \, dx$$

$$\int_0^{\pi/2} \sec^2(x) \, dx$$

$$\int_{-1}^1 \frac{1}{x^6} \, dx$$

$$\int_1^5 \frac{1}{\sqrt{x-1}} \, dx$$

$$\int_0^\infty \frac{8}{(x^2+1)^2} \, dx$$

Determine if the following series converge or diverge. Clearly state which tests you are using and show the necessary work.

$$\sum_{n=8}^{\infty} \frac{n^2}{n^4 + n - 8}$$

$$\sum_{n=1}^{\infty} \frac{n^5}{n^n}$$

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

$$\sum_{n=1}^{\infty} \frac{(\arctan(n))^2}{1 + n^2}$$

$$\sum_{n=3}^{\infty} \frac{\ln(\ln(n))}{n \ln(n)}$$

$$\sum_{n=1}^{\infty} \frac{2^n}{n!}$$

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$$\sum_{n=0}^{\infty} \frac{2^n}{5^n}$$

$$\sum_{n=1}^{\infty} \frac{2^n + n^2}{3^n + n^3}$$

$$\sum_{n=2}^{\infty} \frac{2^n}{(\ln(n))^{n^2}}$$

$$\sum_{n=2}^{\infty} \frac{n^2}{\sqrt{n^6 + 4}}$$

$$\sum_{n=1}^{\infty} \frac{10n^2 + n \sin(n)}{n^4}$$

$$\sum_{n=1}^{\infty} \frac{10n^{107}}{(3.2)^n}$$

$$\sum_{n=1}^{\infty} \frac{n^3 + \sqrt{n}}{2n^3 + 1}$$

Determine if the following series are absolutely convergent, conditionally convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln(n))^2 n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1.02}}$$

For what  $p$  does  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  converge?

For what  $p$  does  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}$  converge?

If any of the following series converges find the sum. If not, explain why.

$$\sum_{n=1}^{\infty} \arctan(n-1) - \arctan(n)$$

$$\sum_{n=5}^{\infty} \frac{3}{\ln(n)} - \frac{3}{\ln(n+1)}$$

$$\sum_{n=1}^{\infty} \frac{1}{2^n}$$

$$\sum_{n=2}^{\infty} \frac{4}{3^{2n}}$$

$$\sum_{n=0}^{\infty} \frac{3^{n+3}}{7^{2n}}$$

$$\sum_{n=8}^{\infty} \sqrt{n+1} - \sqrt{n}$$

$$\sum_{n=0}^{\infty} \frac{2^{n+1} + 3}{5^n}$$

Use geometric series to write the following decimals as a single fraction:

$$.\overline{1}$$

$$1.\overline{14}$$

Give the radius and interval of convergence. On the boundary of the interval of convergence, are there any points of conditional convergence?

$$\sum_{n=1}^{\infty} \frac{(2x)^n}{5^n}$$

$$\sum_{n=1}^{\infty} \frac{(x-4)^n}{n}$$

$$\sum_{n=1}^{\infty} \frac{x^n}{n!}$$

$$\sum_{n=1}^{\infty} \frac{x^n n!}{(2n+1)!}$$

$$\sum_{n=1}^{\infty} \frac{(x+1)^n}{n^2}$$

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{5^n}$$

$$\sum_{n=1}^{\infty} n^2 (x+4)^n$$

Starting with the geometric power series

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots = \sum_{n=0}^{\infty} x^n,$$

give a power series expansion for the following functions, as well as the radius of convergence.

$$\frac{1}{1-2x} \quad \text{centered at } x = 0$$

$$\frac{1}{-x} \quad \text{centered at } x = -1$$

$$\frac{1}{3-12x^2} \quad \text{centered at } x = 0$$

$$\frac{-2x}{(1+x^2)^2} \quad \text{centered at } x = 0$$

$$\arctan(x) \quad \text{centered at } x = 0$$

Give the Taylor or Maclaurin Series for the following functions. Make sure to give the radius of convergence for the series as well.

$$\sin(x) \quad \text{centered at } x = 0$$

$$\cos(x) \quad \text{centered at } x = \pi$$

$$\cos(x) \quad \text{centered at } x = \frac{\pi}{6}$$

$$e^x \quad \text{centered at } x = 2$$

Give the Taylor series for  $e^x$  centered at 0. Use this to find the a power series for  $e^{-x^2}$  THEN find a power series for  $\int e^{-x^2} dx$  (make sure to give the radius of convergence).

Give the Taylor series for  $\sin(x)$  centered at 0. Use this to find the a power series for  $\frac{\sin(x)}{x}$  THEN find a power series for  $\int \frac{\sin(x)}{x} dx$  (make sure to give the radius of convergence).

Calculate the Taylor polynomial of order 3 for  $f(x) = 4^x$  centered at  $a = 0$ .

Calculate the Taylor polynomial of order 2 for  $f(x) = \arctan(x)$  centered at  $a = 1$ .

Calculate the Taylor polynomial of order 3 for  $f(x) = e^{x^2}$  centered at  $a = 1$ .

Using a Taylor polynomial of order 3 centered at 16, approximate  $\sqrt{17}$ .

Using a Taylor polynomial of order 4 centered at 0, approximate  $\cos(\frac{1}{10})$ .

Using a Taylor polynomial of order 4 centered at 1, approximate  $\ln(11/10)$ .

**New Problems Added Aug 7th:**

Use a power series to find the antiderivative of the given function

$$f(x) = \frac{\sin(x)}{x}$$

$$f(x) = \sin(x^5)$$

$$f(x) = e^{x^3}$$