

Name:

Section: 2 4 (circle one)

1. Find the length of the curve

$$\mathbf{r}(t) = e^t \cos(t) \mathbf{i} + e^t \sin(t) \mathbf{j} + e^t \mathbf{k}$$

from

$$t = -\ln(4) \text{ to } t = 0.$$

$$\mathbf{v}(t) = [e^t \sin(t) + e^t \cos(t)] \mathbf{i} + [e^t \cos(t) + e^t \sin(t)] \mathbf{j} + e^t \mathbf{k}$$

$$\begin{aligned} |\mathbf{v}(t)| &= \sqrt{(e^t \cos(t) - e^t \sin(t))^2 + (e^t \cos(t) + e^t \sin(t))^2 + e^{2t}} \\ &= \sqrt{2e^{2t} \cos^2 t + 2e^{2t} \sin^2 t + e^{2t}} \\ &= \sqrt{3e^{2t}} = \sqrt{3} e^t. \end{aligned}$$

So, the length of the curve is

$$\begin{aligned} \int_{-\ln(4)}^0 |\mathbf{v}(t)| dt &= \int_{-\ln(4)}^0 \sqrt{3} e^t dt \\ &= \sqrt{3} \left(e^t \Big|_{-\ln(\frac{1}{4})}^0 \right) \\ &= \sqrt{3} \left(1 - \frac{1}{4} \right) \\ &= \frac{3\sqrt{3}}{4} \end{aligned}$$

2. For the curve

$$r(t) = e^t \cos(t) \mathbf{i} + e^t \sin(t) \mathbf{j} + 2 \mathbf{k}$$

find the unit normal, T , the principle unit normal, N , the curvature, κ , and... find a vector valued function, $B(t)$ such that $B(t)$ is orthogonal to both $T(t)$ and $N(t)$ for all values of t .

$$\mathbf{V}(t) = (\underbrace{e^t \cos(t)}_a - \underbrace{e^t \sin(t)}_b) \mathbf{i} + (\underbrace{e^t \cos t}_a + \underbrace{e^t \sin t}_b) \mathbf{j}$$

$$|\mathbf{V}(t)| = \sqrt{(a-b)^2 + (a+b)^2}$$

$$= \sqrt{2a^2 + 2b^2}$$

$$= \sqrt{2e^{2t} \cos^2 t + 2e^{2t} \sin^2 t}$$

$$= \sqrt{2e^{2t}} = \sqrt{2} e^t.$$

Now, we can compute

$$T = \frac{\mathbf{V}(t)}{|\mathbf{V}(t)|} = \frac{(\cos(t) - \sin(t))\mathbf{i} + (\cos t + \sin t)\mathbf{j}}{\sqrt{2}}$$

$$\text{so that } \frac{dT}{dt} = \frac{(-\sin t - \cos t)\mathbf{i} + (-\sin t + \cos t)\mathbf{j}}{\sqrt{2}}$$

~~and $\mathbf{B}(t) = T(t) \times N(t)$~~

$$\text{Further, } |\frac{dT}{dt}| = \left[\sqrt{(\sin t + \cos t)^2 + (\cos t - \sin t)^2} \right] \frac{1}{\sqrt{2}}$$

$$= \sqrt{2\cos^2 t + 2\sin^2 t} \frac{1}{\sqrt{2}} = \sqrt{2} \cdot \frac{1}{\sqrt{2}} = 1$$

$$\text{so, } N = \frac{\frac{dT}{dt}}{|\frac{dT}{dt}|} = \frac{(-\sin t - \cos t)}{\sqrt{2}} \mathbf{i} + \frac{(\cos t - \sin t)}{\sqrt{2}} \mathbf{j}$$

$$\text{and } K = \frac{1}{|\mathbf{V}|} |\frac{dT}{dt}| = \frac{1}{\sqrt{2} e^t}$$

For $\mathbf{B}(t)$ we can compute

$$\mathbf{T} \times \mathbf{N} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\cos t - \sin t}{\sqrt{2}} & \frac{\cos t + \sin t}{\sqrt{2}} & 0 \\ -\frac{\cos t - \sin t}{\sqrt{2}} & -\frac{\sin t + \cos t}{\sqrt{2}} & 0 \end{vmatrix} = \left[\frac{\cos^2 t - 2\cos t \sin t + \sin^2 t}{2} - \frac{(-\cos^2 t - 2\sin t \cos t - \sin^2 t)}{2} \right] \mathbf{k}$$

$$= \left[\frac{1}{2} + \frac{1}{2} \right] \mathbf{k}$$

$$= \mathbf{k}$$