

Worksheet 7

Solutions / 15 points

Name:

Section: 2 4 (circle one)

1. For $f(x, y) = \frac{\sqrt{x}e^{\cos(x^2y)}}{x+y}$, find the following:

$$\frac{\partial f}{\partial x} = \frac{\left[\frac{1}{2\sqrt{x}} e^{\cos(x^2y)} + \sqrt{x} e^{\cos(x^2y)} (-\sin(x^2y)) \cdot 2xy \right] (x+y) - \sqrt{x} e^{\cos(x^2y)}}{(x+y)^2}$$

$$\frac{\partial f}{\partial y} = \frac{\sqrt{x} e^{\cos(x^2y)} (-\sin(x^2y) \cdot x^2) (x+y) - \sqrt{x} e^{\cos(x^2y)}}{(x+y)^2}$$

Note: we can simplify $\frac{\partial f}{\partial y}$ to $= x^{5/2} \cancel{e^{\cos(x^2y)}} \sin(x^2y)(x+y) - \sqrt{x} e^{\cos(x^2y)}$

$$\frac{\partial^2 f}{\partial y \partial x} = \text{Define } \alpha = -x^{5/2} e^{\cos(x^2y)} \sin(x^2y)(x+y) - \sqrt{x} e^{\cos(x^2y)} \text{ and } \beta = (x+y)^2.$$

↑
↓ The same.

$$\frac{\partial d}{\partial x} = -\frac{5}{2} x^{3/2} \cdot e^{\cos(x^2y)} \sin(x^2y)(x+y) + (-x^{5/2}) [e^{\cos(x^2y)} \cdot (-\sin(x^2y)) 2xy \cdot \sin(x^2y)(x+y) + e^{\cos(x^2y)} \cdot \Phi]$$

$$\text{where } \Phi = \cos(x^2y) \cdot 2xy(x+y) + \sin(x^2y). \text{ Also } \frac{\partial \beta}{\partial x} = 2(x+y).$$

$$\frac{\partial^2 f}{\partial x \partial y} = \text{Now, } \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = \frac{\frac{\partial d}{\partial x} \beta - \alpha \frac{\partial \beta}{\partial x}}{\beta^2}.$$

$$\frac{\partial^2 f}{\partial x^2} =$$

These two are extra credit. You MUST show your work.

$$\frac{\partial^2 f}{\partial y^2} =$$

2. For the function $w = f(x, y, z)$, with $x = g(\alpha, \beta, \gamma)$, $y = h(\alpha, \beta, \gamma)$ and $z = k(\alpha, \beta, \gamma)$, find the following:

$$\frac{\partial w}{\partial \alpha} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial \alpha} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial \alpha} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial \alpha}$$

$$\frac{\partial w}{\partial \beta} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial \beta} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial \beta} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial \beta}$$

$$\frac{\partial w}{\partial \gamma} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial \gamma} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial \gamma} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial \gamma}$$

3. Let $f(x, y) = \sin(x^2 + y^2)$, and $x = \sin(t)e^t$, $y = \cos(t)e^t$. Find $\frac{df}{dt}$.

As we know $\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$, so

$$\begin{aligned}\frac{df}{dt} &= \cos(x^2 + y^2) \cdot 2x (\cos(t)e^t + \sin(t)e^t) + \cos(x^2 + y^2) \cdot 2y (-\sin(t)e^t + \cos(t)e^t) \\ &= \cos(e^{2t}) \cdot 2\sin(t)e^t (\cos(t)e^t + \sin(t)e^t) + \cos(e^{2t}) 2\cos(t)e^t (-\sin(t)e^t + \cos(t)e^t) \\ &= \cos(e^{2t}) \cdot 2e^{2t}\end{aligned}$$

Check: $f(t) = \sin(\sin^2(t)e^{2t} + \cos^2(t)e^{2t}) = \sin(e^{2t})$, so, $f'(t) = \cos(e^{2t}) \cdot 2e^{2t}$.
(This way looks easier!)