

Name:

Solutions

Section: 2 4 (circle one)

1. Let $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 12$.a) Find the critical points of f .

$$f_x = 3x^2 + 6x \quad \text{and} \quad f_y = 3y^2 - 6y$$

\therefore C.P.s are
 $(0,0), (0,2),$
 $(-2,0), (-2,2)$.

If $f_x = 0$, then

$$0 = 3x^2 + 6x \\ = 3x(x+2)$$

$$\Rightarrow x = 0 \text{ or } x = -2$$

If $f_y = 0$, then

$$0 = 3y^2 - 6y \\ = 3y(y-2)$$

$$\Rightarrow y = 0 \text{ or } y = 2$$

b) Find f_{xx} , f_{yy} and f_{xy} .

$$f_{xx} = 6x + 6, \quad f_{yy} = 6y - 6 \quad \text{and} \quad f_{xy} = 0.$$

c) Determine if each critical point is a local min, local max or saddle point.

$$D = f_{xx} f_{yy} - f_{xy}^2 = (6x+6)(6y-6).$$

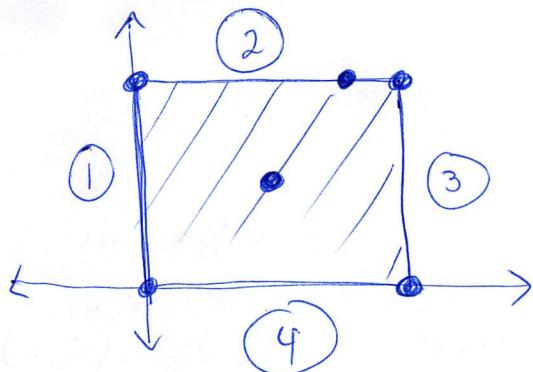
$$D(0,0) = -36 \Rightarrow (0,0) \text{ is a saddle point.}$$

$$D(0,2) = 36, \text{ and } f_{xx}(0,2) = 6 > 0 \therefore (0,2) \text{ is a local min.}$$

$$D(-2,0) = 36 \quad \text{and} \quad f_{xx}(-2,0) = -6 < 0 \therefore (-2,0) \text{ is a local max.}$$

$$D(-2,2) = -36 \Rightarrow (-2,2) \text{ is a saddle point.}$$

2. Find the absolute minimum and maximum of the function $f(x, y) = 48xy - 32x^3 - 24y^2$ on the rectangular region $0 \leq x \leq 1$ and $0 \leq y \leq 1$.



$$f_x = 48y - 96x^2, f_y = 48x - 48y$$

$$f_y = 0 \Rightarrow x = \cancel{48}y,$$

$$\begin{aligned} f_x = 0 \text{ and } x = y &\Rightarrow 0 = 48x - 96x^2 \\ &\Rightarrow x = 0 \text{ or } x = \frac{1}{2} \end{aligned}$$

$$x = \frac{1}{2} \Rightarrow y = \frac{1}{2}, x = 0 \Rightarrow y = 0.$$

So, we have C.P. @ $(\frac{1}{2}, \frac{1}{2})$ and $(0,0)$.

① Along $f(0,y)$, $f(0,y) = -24y^2$, $f'(0,y) = -48y$, so, this gives us a (~~another~~) point to check, $(0,0)$.

② Along $f(x,1)$, $f(x,1) = 48x - 32x^3 - 24$.

$$f'(x,1) = 48 - 96x^2 \Rightarrow x = \pm \sqrt{\frac{1}{2}} \text{ and } x = -\frac{1}{2}$$

is not in our region.

③ Along $f(1,y)$, $f(1,y) = 48y - 32 - 24y^2$, So, we get the point $(\frac{1}{\sqrt{2}}, 1)$

$$f'(1,y) = 48 - 48y \Rightarrow y = 1. \text{ So we get the point}$$

④ Along $f(x,0)$, $f(x,0) = -32x^3 \Leftarrow f'(x,0) = -96x^2 \Rightarrow \frac{(1,1)}{x=0}.$
 $\Rightarrow (0,0)$

In total we have the points

$(0,0)$	\dots	$f(0,0) = 0$
$(1,0)$	\dots	$f(1,0) = -32$
$(1,1)$	\dots	$f(1,1) = -8$
$(\frac{1}{\sqrt{2}}, 1)$	\dots	$f(\frac{1}{\sqrt{2}}, 1) = \frac{32}{\sqrt{2}} - 24$, knowing that $\sqrt{2} \approx 1.4$, $f(\frac{1}{\sqrt{2}}, 1) \approx -1.3$
$(0,1)$	\dots	$f(0,1) = -24$
$(\frac{1}{2}, \frac{1}{2})$	\dots	$f(\frac{1}{2}, \frac{1}{2}) = 2$

\therefore abs min @ $(1,0)$ and
abs max @ $(\frac{1}{2}, \frac{1}{2})$.