Math 203 **Midterm Review**

† **Corrections/alterations have been marked by larger font and a †.**

(1) (10 points)
Using the **limit definition** of the derivative, give $f'(x)$ if

$$f(x) = \frac{1}{x^2+1}$$

$$f(x) = \frac{1}{x-3}$$

$$f(x) = \sqrt{x}$$

$$f(x) = x^2 + 5x$$

(2) (10 points) Compute

$$\lim_{x\to 9} \frac{\sqrt{x-5}-2}{x-9}$$

$$\lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6}$$

$$\lim_{x\to 15} \frac{\sqrt{x+1}-4}{x-15}$$

$$\lim_{x\to 1} \frac{x^2+2x-3}{x-1}$$

$$\lim_{x\to 2} \frac{x^2+x-6}{x-2}$$

$$\lim_{x\to 3} \frac{x^2-9}{x-3}$$

(3) (10 points) Take the derivative of the following, be VERY careful!

$$f(x) = \frac{\sqrt[3]{\sin(x^2)}}{x^2 + 2x + \pi}$$

$$k(x) = \sqrt{\cos(\sqrt{\sin(\sqrt{\ln(x)e^x}}))}$$

$$g(x) = \frac{\cos(x^4)10x^2}{x^2 + x - 4}$$

$$h(x) = \frac{\sqrt{\sin(2x^5)}x^2}{e^{x^2}}$$

$$m(x) = \frac{\sin(x)}{\cos(x)}$$

(4) (10 points)

Graph the following functions, you will be asked to include and any asymptotes (slant, horizontal and vertical), local min/max (if there is any), intervals of increase/decrease, intervals of concavity and, of course, a sketch that demonstrates all of these characteristics.

$$h(x) = \frac{x^2 - 1}{x + 2}$$

$$j(x) = \frac{x^2 - 1}{x}$$

$$k(x) = \frac{x^2 - 4}{x + 3}$$

$$(\dagger) l(x) = \frac{x^2 - 4}{x^2}$$

$$(\dagger) m(x) = \frac{x^2 - 2x + 1}{x^2}$$

Draw a function with the following properties: $f(2) = 1$, $f'(2) = f'(4) = f'(-2) = 0$, $f''(3) = f''(0) = f''(5) = 0$, $f'(x) > 0$ on $(-\infty, -2) \cup (2, 4)$, $f'(x) < 0$ on $(-2, 2) \cup (4, \infty)$, $f''(x) > 0$ on $(0, 3) \cup (5, \infty)$, $f''(x) < 0$ on $(-\infty, 0) \cup (3, 5)$ and $\lim_{x \to \infty} f(x) = 0$ (note that this means that there is a horizontal asymptote at $y = 0$).

(5) (10 points)

Suppose that we are selling manapuas. Suppose that if we want to sell 1000 manapuas, then we can sell them at 1 dollar each, and if we only want to sell 400 manapuas, we can sell them at 1.75 dollars. If demand is linear, give the demand equation and give how many manapuas should we make and how much do we sell them for if revenue is maximized?

Suppose that we are selling gondolas. Suppose that if we want to sell 10000000 gondolas, then we can sell them at 10 dollars each, and if we only want to sell 4000 gondolas, we can sell them at 1750 dollars. If demand is linear, give the demand equation and give how many gondolas should we make and how much do we sell them for if revenue is maximized?

Suppose that we are selling tacos. Suppose that if we want to sell 5000 tacos, then we can sell them at 1 dollar each, and if we only want to sell 4000 tacos, we can sell them at 1.75 dollars. If demand is linear, give the demand equation and give how many tacos should we make and how much do we sell them for if revenue is maximized?

(6) (10 points) Show that the given point is indeed on the graph. Give the equation of the tangent line at the given point (if one exists).

$$\sin(x) + \cos(x) - \sin(yx) = 1 \quad P = (0,0)$$

$$y^5x^2 - 2\frac{x}{y} = -1 \quad P = (1,1)$$

$$y^4x - 4y^2x = -3 \quad P = (1,1)$$

(7) (10 points) See handout of "optimization problems"

(8) (10 points) Evaluate the indefinite integrals:

$$\int x^5 + 10x^2 + 4 \;\; dx$$
$$\int \frac{\sqrt{x}+4}{2\sqrt{x}} \;\; dx$$
$$\int \cos(x^2)2x \;\; dx$$
$$\int x^3 + 10x^2 \;\; dx$$

(9) (10 points) Evaluate the definite integrals:

$$\int_{-1}^{1} x^5 + 10x^2 + 4 \;\; dx$$
$$\dagger \int_{1}^{2} \frac{\sqrt{x}+4}{2\sqrt{x}} \;\; dx$$
$$\int_{0}^{\sqrt{\pi}} \cos(x^2)2x \;\; dx$$
$$\int_{0}^{2} x^3 + 10x^2 \;\; dx$$

(10) (10 points) Find the Area between the given function and the x-axis on the given interval.

$$f(x) = x^2 - 4 \quad \text{on} \quad [0,4]$$
$$f(x) = x^2 - 3x + 4 \quad \text{on} \quad [0,6]$$
$$f(x) = x^2 - 9 \quad \text{on} \quad [0,4]$$
$$f(x) = \sin(x) \quad \text{on} \quad [0, \frac{3\pi}{2}]$$
$$f(x) = x^2 - x - 2 \quad \text{on} \quad [0,4]$$

(11) (10 points) Find the area between the given curves on the interval $[-1,1]$

$$f(x) = x^2, \quad g(x) = x^3$$
$$f(x) = x, \quad g(x) = x^4$$
$$f(x) = 2x, \quad g(x) = 2x^2$$
$$f(x) = x^4, \quad g(x) = x^3$$