Math 241 **Midterm Review**

† **Corrections/alterations have been marked by larger font and a †.**

(1) (10 points)

Using the **limit definition** of the derivative, give $f'(x)$ if

$$f(x) = \frac{1}{x^2+1}$$

$$f(x) = \frac{1}{x-3}$$

$$f(x) = \sqrt{x}$$

$$f(x) = x^2 + 5x$$

(2) (10 points) Compute

$$\lim_{x\to 9} \frac{\sqrt{x-5}-2}{x-9}$$

$$\lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6}$$

$$\lim_{x\to 15} \frac{\sqrt{x+1}-4}{x-15}$$

$$\lim_{x\to 1} \frac{x^2+2x-3}{x-1}$$

$$\lim_{x\to 2} \frac{x^2+x-6}{x-2}$$

$$\lim_{x\to 3} \frac{x^2-9}{x-3}$$

(3) (10 points) Take the derivative of the following, be VERY careful!

$$f(x) = \frac{\sqrt[3]{\sin(x^2)}}{x^2 + 2x + \pi}$$

$$k(x) = \sqrt{\cos(\sqrt{\sin(\sqrt{\ln(x)e^x}))}}$$

$$g(x) = \frac{\cos(x^4)10x^2}{x^2 + x - 4}$$

$$h(x) = \frac{\sqrt{\sin(2x^5)}x^2}{e^{x^2}}$$

$$m(x) = \frac{\sin(x)}{\cos(x)}$$

(4) (10 points)

Graph the following functions, you will be asked to include and any asymptotes (slant, horizontal and vertical), local min/max (if there is any), intervals of increase/decrease, intervals of concavity, **one sided limits at the vertical aymptotes** and of course, a sketch that demonstrates all of these characteristics.

$$h(x) = \frac{x^2 - 1}{x + 2}$$

$$j(x) = \frac{x^2 - 1}{x}$$

$$k(x) = \frac{x^2 - 4}{x + 3}$$

$$(\dagger) l(x) = \frac{x^2 - 4}{x^2}$$

$$(\dagger) m(x) = \frac{x^2 - 2x + 1}{x^2}$$

Draw a function with the following properties: $f(2) = 1$, $f'(2) = f'(4) = f'(-2) = 0$, $f''(3) = f''(0) = f''(5) = 0$, $f'(x) > 0$ on $(-\infty, -2) \cup (2, 4)$, $f'(x) < 0$ on $(-2, 2) \cup (4, \infty)$, $f''(x) > 0$ on $(0, 3) \cup (5, \infty)$, $f''(x) < 0$ on $(-\infty, 0) \cup (3, 5)$ and $\lim_{x \to \infty} f(x) = 0$ (note that this means that there is a horizontal asymptote at $y = 0$).

(5) (10 points)

Using the IVT and MVT:

State the IVT.

(†) **Use the IVT to show that** $f(x) = \sin(x)\cos(x)$ **has a root on** $[-\frac{\pi}{4}, \frac{\pi}{4}]$**. Is this root unique?**

Use the IVT to show that $f(x) = x^3 + 2x + 1$ has a root on $[-1, 1]$. Is this root unique?

State the MVT.

Use the MVT to show that $|\cos(x) - \cos(y)| \leq |x - y|$ (you may use the fact that $|\sin(z)| \leq 1$ for any $z \in \mathbb{R}$.)

Use the MVT to show that any differentiable function, $f(x)$, whose derivative is bounded by 1 enjoys the following property $|f(x) - f(y)| \leq |x - y|$.

(6) (10 points) Show that the given point is indeed on the graph. Give the equation of the tangent line at the given point (if one exists).

$$\sin(x) + \cos(x) - \sin(yx) = 1 \quad P = (0, 0)$$

$$y^5x^2 - 2\frac{x}{y} = -1 \quad P = (1, 1)$$

$$y^4x - 4y^2x = -3 \quad P = (1, 1)$$

(7) (10 points) See handout of "optimization problems" for examples
(8) (10 points) See handout of "Related Rates" for examples
(9) (10 points) Evaluate the indefinite integrals:

$$\int x^5 + 10x^2 + 4 \ \ dx$$
$$\int \frac{\sqrt{x}+4}{2\sqrt{x}} \ \ dx$$
$$\int \cos(x^2)2x \ \ dx$$
$$\int x^3 + 10x^2 \ \ dx$$

(10) (10 points) Evaluate the definite integrals:

$$\int_{-1}^{1} x^5 + 10x^2 + 4 \ \ dx$$
$$\dagger \int_{1}^{2} \frac{\sqrt{x}+4}{2\sqrt{x}} \ \ dx$$
$$\int_{0}^{\sqrt{\pi}} \cos(x^2)2x \ \ dx$$
$$\int_{0}^{2} x^3 + 10x^2 \ \ dx$$
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(x) \ \ dx$$
$$\int_{0}^{1} x^3 + x^2 + x + 1 \ \ dx$$

(11) (10 points) Find the following derivatives

$$\frac{d}{dx}\left(\int_{2}^{\cos(x)} \sqrt{t^2 - t^3}dt\right) =$$
$$\frac{d}{dx}\left(\int_{x^4}^{\cos(x)} \sqrt{2t - t^3}dt\right) =$$
$$\frac{d}{dx}\left(\int_{x^2}^{\cos(x)} \sqrt{10t^{100} - t^3}dt\right) =$$

(12) **(10 points) Find the Area between the given function and the x-axis on the given interval.**

$$f(x) = x^2 - 4 \quad \text{on} \quad [0,4]$$
$$f(x) = x^2 - 3x + 4 \quad \text{on} \quad [0,6]$$
$$f(x) = x^2 - 9 \quad \text{on} \quad [0,4]$$

$$f(x) = \sin(x) \quad \text{on} \quad [0, \frac{3\pi}{2}]$$
$$f(x) = x^2 - x - 2 \quad \text{on} \quad [0, 4]$$

(13) **(10 points) Find the area between the given curves on the interval** $[-1, 1]$

$$f(x) = x^2, \quad g(x) = x^3$$
$$f(x) = x, \quad g(x) = x^4$$
$$f(x) = 2x, \quad g(x) = 2x^2$$
$$f(x) = x^4, \quad g(x) = x^3$$