§ 5.1

1. true
2. true
3. false, 2
4. true
5. false
6. false
7. true
8. true
9. false
10. true
11. false
12. true

27. 2, 3 — no others since two consecutive must be even-odd or odd-even, and 2 is the only even prime.

28. no — any list of three consecutive integers must have one or two evens. The only even prime is 2, so the only possibility is 1, 2, 3, but 1 is not prime.

29. it must be 0 (since the number is divisible by 0 also).
69. Any three consecutive natural numbers include at least one which is a factor of two and exactly one which is a factor of three. So the product has a factor of 2 and 3, so it has a factor of 6, i.e. it is divisible by 6.

71. \( n^2 - n + 41 \) for \( n = 41 \)

\[
41^2 - 41 + 41 = 41^2 \text{ cannot be prime since it factors into } 41 \cdot 41
\]

(can also multiply it out to see)

72. Formula: \( n^2 - n + 41 \)

(a) \( n = 42 \), \( 42^2 - 42 + 41 \)

\[
= 1763 \quad \text{(note: } 1763 = 41 \cdot 43, \text{ not prime)}
\]

(b) \( n = 43 \), \( 43^2 - 43 + 41 \)

\[
= 1847 \quad \text{(prime)}
\]
1. true
2. false
3. true
4. false (5! not prime)
5. false (Remainder says yes)
6. true
7. \[ 1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 = 496 \] true
8. \[ 1 + 2 + 4 + 8 + 16 + 32 + 64 + 127 + 854 + 508 + 1016 + 2022 + 4064 = 8128 \] false
9. \[ 2^{13} - 1 = 8191 \]

8191 is prime

since \( 2^{13} - 1 \) is prime,

\[ 2^{13} - 1 (2^{13} - 1) = 2^{12} (2^{13} - 1) \]

is perfect

= 33550336

10. \ Korea

11. Since \( 2^{30402457} - 1 \) is prime,

\[ 2^{30402456} (2^{30402456} - 1) \] is perfect

12. \[ 496 = 1 + 2 + 3 + \ldots + k \] for some \( k \) — what is it?

To make this fast, can use fact that \( 1 + 2 + 3 + \ldots + k = \frac{k(k+1)}{2} \)

 Either test values for \( k \) or solve the equation

\[ 496 = \frac{k(k+1)}{2} \]

\[ 992 = k(k+1) \]

\[ 0 = k^2 + k - 992 = (k+32)(k-31) \] and since \( k > 0 \), \( k = 31 \)
25. add up: \[ 1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 = 1023 \]

and \[ 1 + 2 + 5 + 10 + 22 + 55 + 110 + 121 + 242 + 605 = 1184 \]

so they are friendly

27. \[ 14 = 3 + 11 \]

28. \[ 22 = 3 + 19 \]

31. \[ 11 = a + 2b, \quad a, b \text{ prime?} \]

\[ 11 = 7 + 4 \]

\[ = 7 + 2 \cdot 2 \text{ and } 2, 7 \text{ are both prime} \]

32. \[ 6 = 11 - 5 \]

\[ 12 = 17 - 5 \]

\[ 18 = 23 - 5 \]