

## WHICH METHODS SATISFY OR VIOLATE WHICH CRITERIA?

Recall that the four fairness criteria are majority, Condorcet, monotonicity, and independence of irrelevant alternatives. Also recall that for a method to satisfy a fairness criterion, every possible election must have a fair result with respect to this notion of fairness—to justify this a logical argument must be made. For a method to violate a fairness criterion, we need only present a single election whose outcome is unfair with respect to this criterion.

**Sample elections.** We will use these elections to demonstrate the various violations.

*Election 1*

	49	48	3
1st	A	B	C
2nd	B	C	B
3rd	C	A	A

*Election 2*

	51	49
1st	A	B
2nd	B	C
3rd	C	A

*Election 3*

	14	8	4
1st	A	B	D
2nd	B	D	A
3rd	C	C	C
4th	D	A	B

*Election 4*

	5	4	3	1
1st	A	C	B	B
2nd	B	B	C	A
3rd	C	A	A	C

*Election 5*

	49	48	3
1st	A	B	C
2nd	B	C	A
3rd	C	A	B

*Election 6*

	10	7	5	5	4
1st	A	D	B	C	B
2nd	C	B	C	D	C
3rd	B	A	A	A	D
4th	D	C	D	B	A

*Election 7*

	4	5	6	2
1st	A	B	C	A
2nd	D	C	A	C
3rd	B	A	D	D
4th	C	D	B	B

*Election 8*

	9	11	7	6	3
1st	A	B	C	A	D
2nd	D	C	A	C	C
3rd	B	A	D	D	B
4th	C	D	B	B	A

### Plurality method.

- Satisfies majority criterion: a majority candidate has the most first-place votes.
- Satisfies monotonicity: if in a reelection the votes change only to favor the previous winner, there can only be more first-place votes for the candidate that already had most of the first-place votes.
- Violates the Condorcet criterion: in Election 1, *B* is a Condorcet candidate yet loses the election by plurality.
- Violates IIA: in Election 1, *A* is the winner by plurality, but if *C* is eliminated then *B* wins the recount.

### **Borda count method.**

- Satisfies monotonicity: if in a reelection the votes change only to favor the previous winner, there can only be more points for the candidate that already had the most points.
- Violates the Condorcet criterion: in Election 2,  $A$  is the Condorcet candidate but  $B$  is the winner of the election.
- Violates majority criterion: in Election 2,  $A$  is the majority candidate but  $B$  is the winner of the election.
- Violates IIA: in Election 3,  $B$  wins by the Borda count method, but if  $C$  is eliminated then  $A$  wins the recount.

### **Instant runoff voting / plurality with elimination.**

- Satisfies majority criterion: a majority candidate wins in the first round.
- Violates the Condorcet criterion: in Election 6,  $D$  is the winner by this method, but  $B$  is a Condorcet candidate.
- Violates monotonicity: in Election 7,  $C$  is the winner by this method, but if in a reelection the two voters in the last column switch their votes and move  $C$  ahead of  $A$ , the winner of the reelection is  $B$ .
- Violates IIA: in Election 4,  $A$  is the winner by this method (in fact both  $B$  and  $C$  are eliminated in the first round), but if  $C$  is eliminated then  $B$  wins the recount. Another example: in Election 5,  $A$  is the winner, but if  $B$  eliminated then  $C$  wins the recount.

### **Pairwise comparisons.**

- Satisfies majority criterion: a majority candidate will win every pairwise (head-to-head) comparison.
- Satisfies the Condorcet criterion: by definition, a Condorcet candidate is one that wins every head-to-head comparison, so this candidate always wins by pairwise comparisons.
- Satisfies monotonicity: if in a reelection the votes change only to favor the previous winner, there can only be more head-to-head wins for the candidate that already had the most head-to-head wins.
- Violates IIA: in Election 8,  $A$  is the winner by this method, but if  $D$  is eliminated then  $B$  wins the recount.