Babylonian Problems with Corrections for Midterm

Instructor : Monique Chyba
Email: chyba@hawaii.edu

Problem 1: Express the numbers 76234, 1265, and 87.432 in sexagesimal.

Let us take the number 76234. You divide this number by 3600 : $76234/3600 = 21.176111111$. You have that $76234 = 21 * 3600 + 634$. The remainder is 634. You divide 634 by 60 and you get $634/60 = 10.56666666$. You have that $634 = 10 * 60 + 34$. You have obtained that 76234=21,10,34.

Let us take the number 1265. You have that $1265/60 = 21.083333$, and $1265 = 21 * 60 + 5$. You have obtained that 1265=21,5.

Let us take the number 87.432. First we deal with the entire part. We have that 87=60+27. then we deal with the decimal part. You take 0.432 and you multiply it by 60: 0.432*60=25.92. Yu take the remainder and you multiply is by 60: 0.92*60=55.2. You take the remainder and you multiply by 60: 0.2*60=12. There is no remainder, therefore the algorithm ended. You obtained that 87.432=1,27;25,55,12 (for the decimal part you keep only the entire part of the multiplications you do in the algorithm until you reached the stage when there is no remainder).

Problem 2: Compute the product $1, 23 * 2, 9$.

To solve tho problem you have two options. Option one is to convert to a number in base 10 and then do the multiplication. Do not forget to convert the number you find back in base 60. The other option is detailed below.

You first write $1,23 = 1 * 60 + 23$ and $2,9 = 2 * 60 + 9$. You introduce $x = 60$, which gives you $1,23 = x + 23$ and $2,9 = 2 * x + 9$. You multiply $(x+23) * (x+9) = x^2 + 32 * x + 207$. You have that $207 = 3*60+27 = 3*x+27$. Hence, $(x+23)*(x+9) = x^2 + 35 * x + 27 = 1,35,27$.

Problem 3: Solve the following system ala the Babylonian false position method. State clearly what steps you are taking

$$2x + 3y = 1600$$
$$5x + 4y = 2600$$

You select the "false" value $\hat{x} = \hat{y}$ which implies by the first equation $5\hat{x} = 1600$ and therefore $\hat{x} = 320$. You then modify the false values as follow: $x = \hat{x} + 3d = 320 + 3d$ and $y = \hat{y} - 2d = 320 - 2d$. Remark the coefficients in front of the d variable come from equation 1 of the system we are trying to solve (so that the d cancel out when replaced in equation 1). The second equation of our system becomes:

$$5x + 4y = 5(320 + 3d) + 4(320 - 2d) = 2880 + 7d = 2600.$$

By solving you obtain $d = -40$ and then $x = 200$, $y = 400$.

Problem 5: Modify the Babylonian root finding method (for $\sqrt{2}$) to find the square root of any number. Use your method to approximate $\sqrt{3}$. Begin with $x_0 = 1$.

You modify the algorithm as follows.

You set $x_0 = 1$. Then, you define $x_1 = 1/2 * (x_0 + 3/x_0) = 2$. And you continue in an iterative way. $x_2 = 1/2 * (x_1 + 3/x_1) = 1.75$, $x_3 = 1/2 * (x_2 + 3/x_2) = 1.73$ etc until you reach the precision asked in the problem (it will be in terms of decimals). The general algorithm is therefore $x_n = 1/2 * (x_{n-1} + 3/x_{n-1})$.