Class 2: **Friday January 22**

TITLE Egyptian and Babylonian mathematics
CURRENT READING: Katz, §1.1-1.2

Homework for Friday January 29
Katz, p. 28-29. #2, #5, #10, #17, #20.

SUMMARY
In today's class we will explore how ancient Egyptians and Babylonians represented numbers and did arithmetic.

Egyptian Mathematics

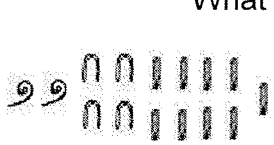
. They used base 10 (decimal).
. Their number system was NOT positional.
. They only allowed unit fractions except for 2/3 and 3/4.
. They could solve 2 equations in 2 unknowns
. They well understood the distribution properties of multiplication and addition.
. They were well-versed in arithmetic operations with fractions
. Possessed two number systems: hieroglyphic and hieratic

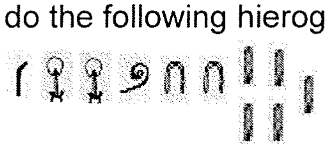
Egyptian Hieroglyphic Numbers

 = 1	 = 1,000	 = 1,000,000
 = 10	 = 10,000	
 = 100	 = 100,000	

1 = vertical stroke	1,000 = lotus flower	1,000,000 = a kneeling figure
10 = heal bone	10,000 = a bent finger	
100 = a snare	100,000 = a burbot fish	

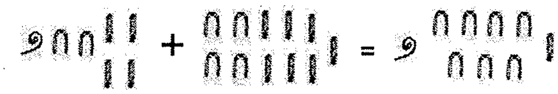
EXAMPLE What numbers do the following hieroglyphics represent?

 =

 =

 =

Addition is executed by grouping



Exercise

What sum is represented by the figures above?

Multiplication and **Division** are both binary

Example: Multiply: 47×24

47	$\times$	24	
47		1	doubling process
94		2	
188		4	
376		8	*
752		16	*

Selecting 8 and 16 (i.e. $8 + 16 = 24$), we have

$$\begin{aligned} 24 &= 16 + 8 \\ 47 \times 24 &= 47 \times (16 + 8) \\ &= 752 + 376 \\ &= 1128 \end{aligned}$$

Example: $329 \div 12$

329	$\div$	12		
12		1	doubling	329
24		2		-192
48		4		137
96		8		-96
192		16		41
384		32		-24
				17
				-12
				5

Now

$$\begin{aligned} 329 &= 16 \times 12 + 8 \times 12 + 2 \times 12 + 1 \times 12 + 5 \\ &= (16 + 8 + 2 + 1) \times 12 + 5 \end{aligned}$$

So,

$$329 \div 12 = 27\frac{5}{12} = 27 + \frac{1}{3} + \frac{1}{12}.$$

Fractions

Fractions were written with a symbol, called the **horus-eye**, in the numerator over the hieroglyphics for the number in the denominator. In hieratic, a dot was placed over the hieratic symbol for the number in the denominator.

Examples

1/3 1/10 1/25

There were special symbols for ½, 2/3 and 3/4

1/2 2/3 3/4

The Katz book uses $\overline{n}$ to represent 1/n and $\bar{\bar{3}}$ to represent 2/3

Babylonian Mathematics
. They used no zero.
. More general fractions, though not all fractions, were admitted.
. They could extract square roots.
. They could solve linear systems.
. They worked with Pythagorean triples.
. They solved cubic equations with the help of tables.
. They studied circular measurement.
. Their geometry was sometimes incorrect.

For enumeration the Babylonians used symbols for 1, 10, 60, 600, 3,600, 36,000, and 216,000.
Below are four of the symbols..

Cuneiform numerals

They did arithmetic in base 60, ***sexagesimal***
For our purposes we will use just the first two symbols

∨ = 1 ≺ = 10

All numbers will be formed from these.

Example:

≺≺ ∨∨∨
≺≺≺ ∨∨∨∨ = 57

Note the notation was **positional** and **sexagesimal**:

≺≺ ≺≺ $= 20 \cdot 60 + 20$ and ∨∨ ∨∨ ≺ ∨ $=2\cdot 60^2+2\cdot 60+11=7{,}331$

Notation
We will write these numbers as **20,20** and **2,2,11**
In general
It should be noted that Babylonians did not use a zero but just left a space if a number was missing a particular power (Katz, 12).

Why Base 60?
Many different reasons were given for this choice, some are:
1. The number of days, 360, in a year gave rise to the subdivision of the circle into 360 degrees, and that the chord of one sixth of a circle is equal to the radius gave rise to a natural division of the circle into six equal parts. This in turn made 60 a natural unit of counting.
2. The Babylonians used a 12 hour clock, with 60 minute hours.
3. The base 60 provided a convenient way to express fractions from a variety of systems as may be needed in conversion of weights and measures.
4. The number 60 is the product of the number of planets (5 known at the time) by the number of months in the year, 12.
5. The combination of the duodecimal system (base 12) and the base 10 system leads naturally to a base 60 system.

Babylonian Computations

Addition

Exercise

Show that 23,37+41,32=1,5,9

Multiplication

Exercise

Show that 34 X 9 =5,6

Division and Fractions

The Babylonians only dealt with **regular** sexagesimal numbers, that is numbers whose reciprocal is a terminating sexagesimal fraction (Katz, 14).

EXAMPLE

$$\frac{1}{6} = \frac{10}{60} = ; \prec$$

$$\frac{1}{9} = ; \begin{matrix} \vee\vee\vee & \prec\prec \\ \vee\vee\vee' & \prec\prec \end{matrix}$$

Exercise

How would you compute 1/8 as a sexigesimal fraction?

These reciprocals were often calculated with the assistance of tables.
A table of all products equal to sixty has been found.

2	30	16	3, 45
3	20	18	3,20
4	15	20	3
5	12	24	2,30
6	10	25	2,25
8	7,30	27	2,13,20
9	6,40	30	2
10	6	32	1;52,30
12	5	36	1,40
15	4	40	1,30

From the table we can see that

$$8 \times 7;30 = 8 \times (7 + \frac{30}{60}) = 60$$

It can also be used to compute reciprocals

$$\frac{1}{8} = 0;7,30 = \frac{7}{60} + \frac{30}{60^2}$$

Exercise

What would an equivalent table look like for our decimal system?
(HINT: You only keep fractions which have terminating decimal representations)