

Linear Thinking

Solving First Degree Equations

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MAT 400

Chessa Horomanski

Jessica DiPaul



The Rhind Papyrus

- Named after A. Henry Rhind
- Collection of problems probably used for training young scribes in Ancient Egypt



Source: HistoryofScience.com



Sample Problem

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A quantity; its half and its third are added to it. It becomes ten.

- For us: $x + \frac{1}{2}x + \frac{1}{3}x = 10$
- A scribe is instructed to solve it as we would:

Divide 10 by $1 + \frac{1}{2} + \frac{1}{3}$



A Different Method

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Example:

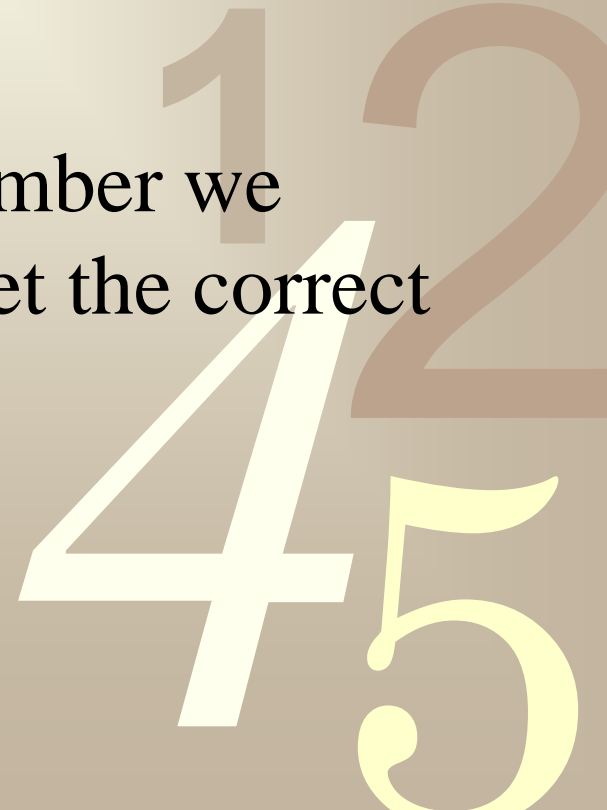
A quantity; its fourth is added to it. It becomes fifteen.

- Assume (posits) the quantity is 4
- Take 4 and add its fourth to it, you get $4+1=5$, but we wanted 15
- We need to multiply what we got (5) by 3 to get what we wanted to get (15)
- Take our guess (4) and multiply by 3 to get an answer of 12

False Position

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- We posit an answer that we don't really expect to be the right one
 - Makes the computations easy
- Use the incorrect result find number we need to multiply our guess to get the correct answer



You try it!

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Use the method of false position to solve this problem.

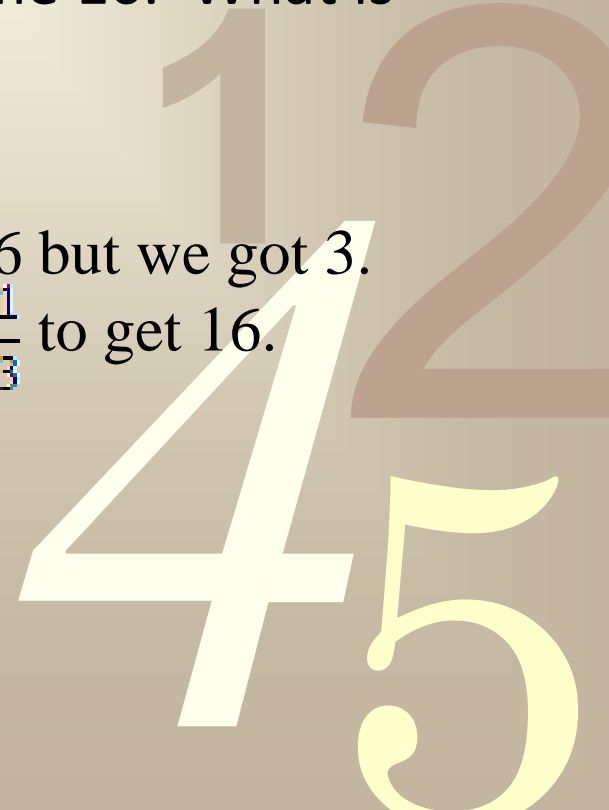
A quantity and its $\frac{1}{2}$ added together become 16. What is the quantity?

Assume 2. Take $2 + \frac{1}{2} \cdot 2 = 3$. We wanted 16 but we got 3.

We need to multiply what we got (3) by $5\frac{1}{3}$ to get 16.

Take our guess 2 and multiply it by $5\frac{1}{3}$

$$5\frac{1}{3} \cdot 2 = 10\frac{2}{3} \leftarrow \text{The quantity}$$



Symbols

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- Using symbols

$$Ax = B$$

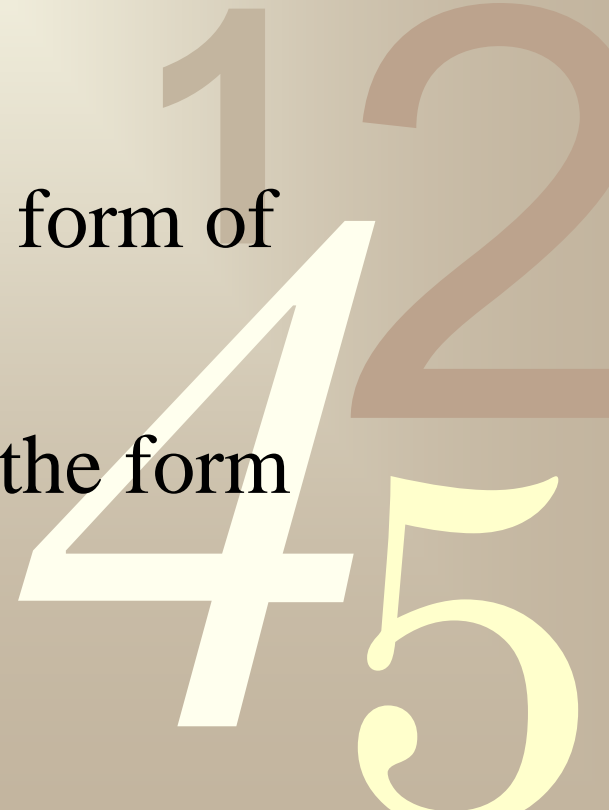
$$A(kx) = k(Ax) = kB$$

- Only works on equations in the form of

$$Ax = B$$

- Does not work on equations of the form

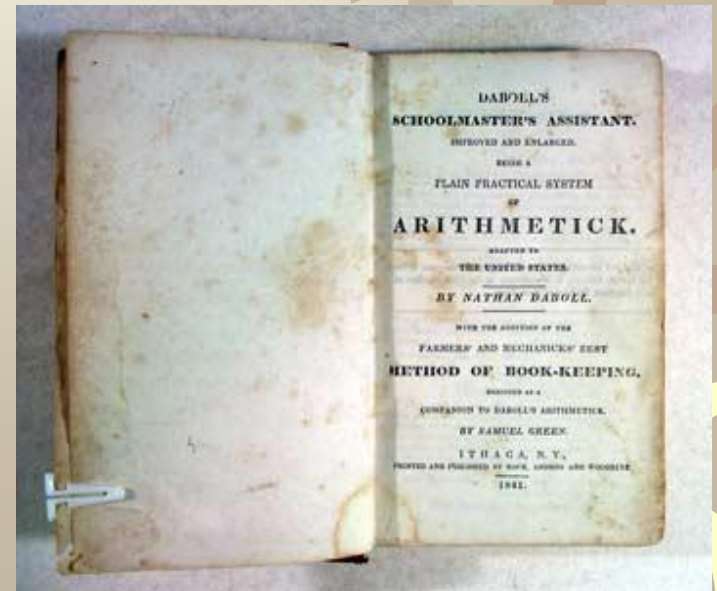
$$Ax + C = B$$



Double False Position

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- Effective method for solving linear equations that it continued to be used long after the invention of algebraic notations
- Requires no algebra
- Taught in arithmetic textbooks
 - *Daboll's Schoolmaster's Assistant*



Source: HarvestofHistory.org

Example of Double False Position

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A purse of 100 dollars is to be divided among four men A, B, C and D, so that B may have four dollars more than A, and C eight dollars more than B, and D twice as many as C. What is each one's share of the money?



Modern Approach:

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Let x be the amount given to A. Then B gets $x+4$, C gets $(x+4)+8=x+12$ and D gets $2(x+12)$. Total is \$100.

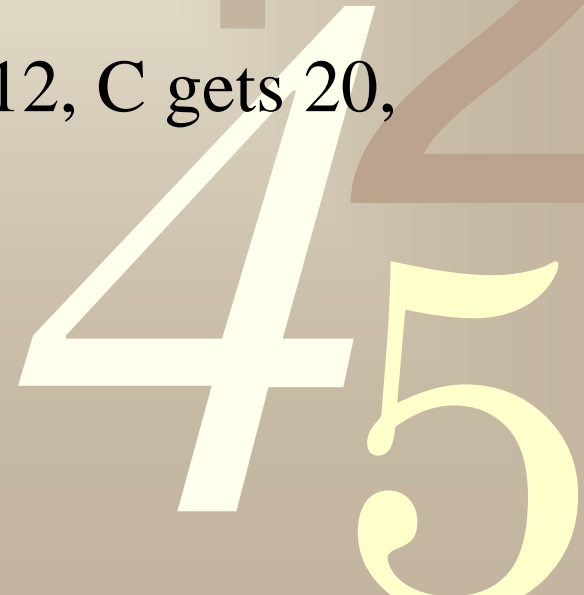
$$x + (x + 4) + (x + 12) + 2(x + 12) = 100$$



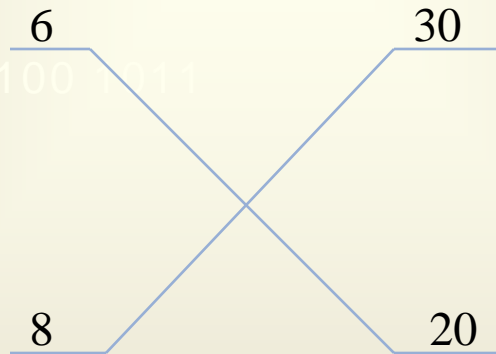
Daboll's Method

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- Make a guess – assume A gets 6 dollars
 - Then B gets 10, C gets 18, and D gets 36
 - Adding them together we get \$70, only \$30 off
- Let's try again
 - Assume A gets 8 dollars, B gets 12, C gets 20, and D gets 40
 - Total now is \$80, still off by \$20



MAGIC!!



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- Cross multiply and subtract the differences to get 120
- Divide the difference of errors to get 10
- Divide 120 by 10 so the right choice for man A is \$12
- Only works when both errors are same type

Isn't this puzzling?? Why does it work?

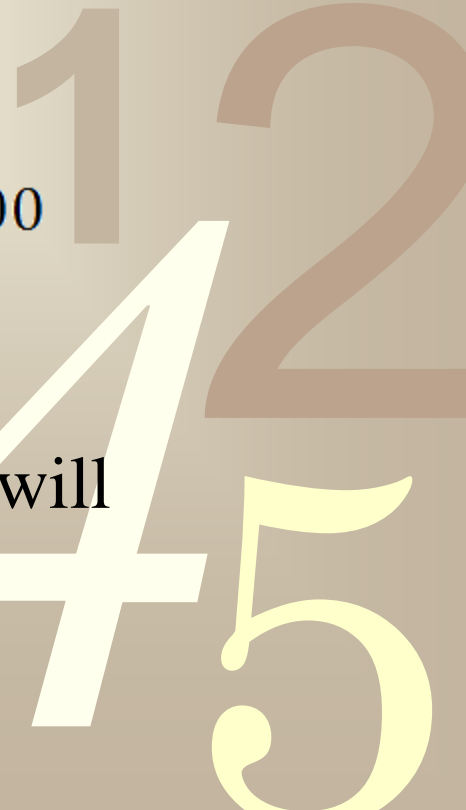
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Let's use some graphical thinking.

$$x + (x + 4) + (x + 12) + 2(x + 12) = 100$$



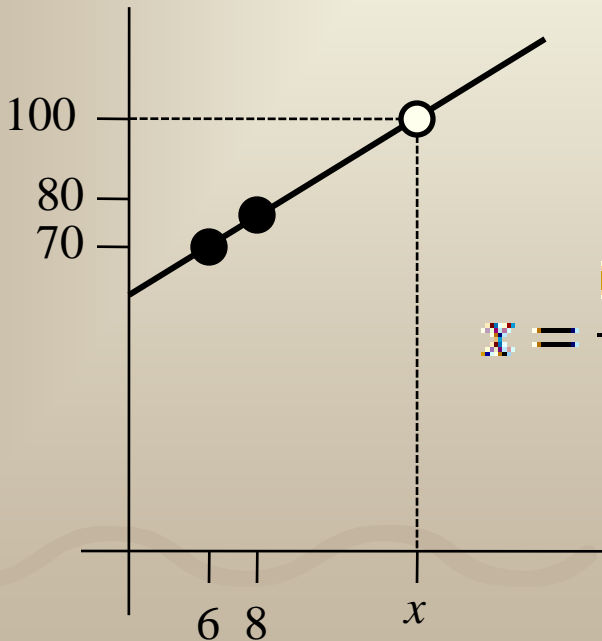
By simplifying the left side, the equation will be something of the form $mx + b = 100$



We need 2 points.

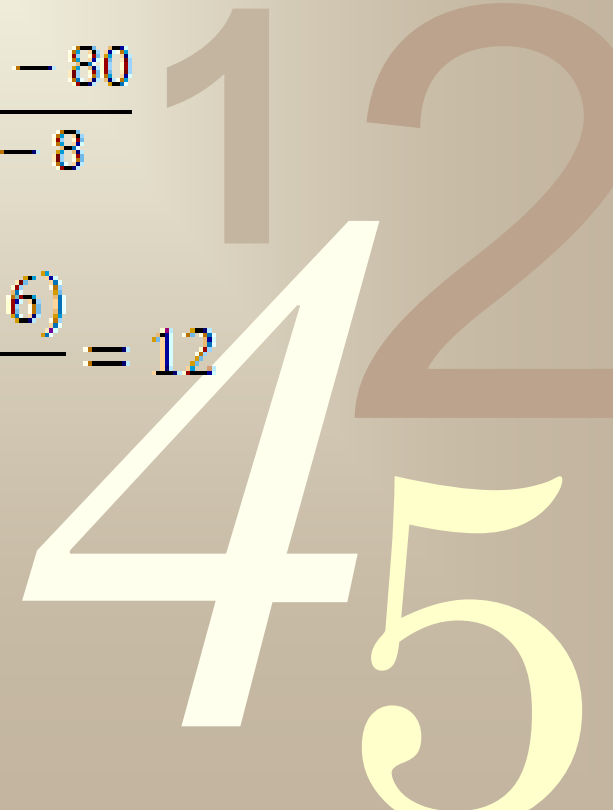
We will use our guesses: (6, 70) and (8, 80)

We want to find x so that $(x, 100)$ is on the same line.




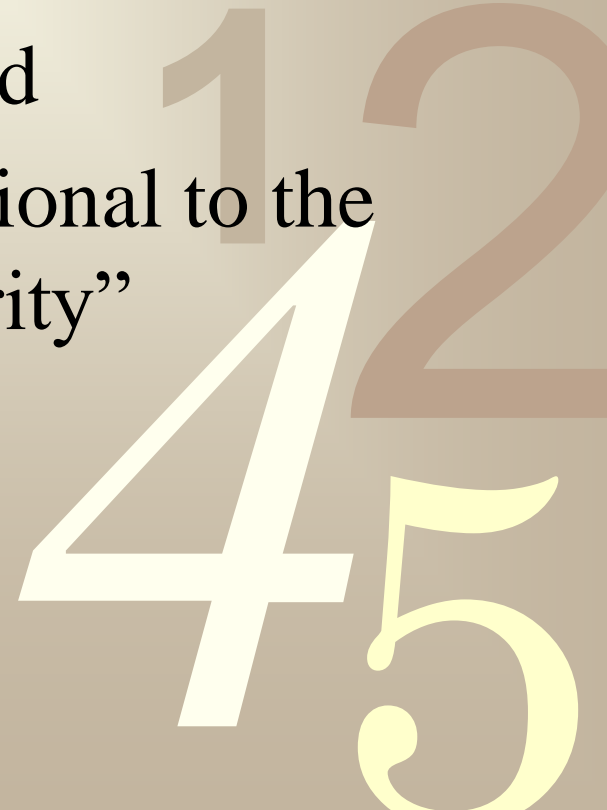
$$\frac{100 - 70}{x - 6} = \frac{100 - 80}{x - 8}$$

$$x = \frac{(30 \times 8) - (20 \times 6)}{30 - 20} = 12$$



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- Our way of understanding equations as lines is quite recent (17th century)
- Double false position is very old
- Change in the output is proportional to the change in the input  “linearity”



“Linear” and “Nonlinear”

Linear

Simple relation – a constant ratio – between changes in the input and output

Nonlinear

No such simple relation – very small changes in the input may produce huge changes in the output

- We use linear problems to find approx. solutions to nonlinear ones.
- The methods we use are based on the fundamental insight which served as the basis for the method of false position.

Timeline

1650 B.C.

- Rhind Papyrus
 - False position
 - Double false position

Early 17th century

- Modern way of understanding equations as lines

Early 1800s

- “*Daboll’s Schoolmaster’s Assistant*” –most popular arithmetic book in America prior to 1850

References

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