Preface

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you've reached the level of working the harder problems then you will probably already understand the basics fairly well and won't need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven't been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

Business Applications

1. A company can produce a maximum of 1500 widgets in a year. If they sell x widgets during the year then their profit, in dollars, is given by,

$$P\left(x\right) = 30,000,000 - 360,000x + 750x^2 - \tfrac{1}{3}x^3$$

How many widgets should they try to sell in order to maximize their profit?

Step 1
Because these are essentially the same type of problems that we did in the Absolute Extrema section we will not be doing a lot of explanation to the steps here. If you need some practice on absolute extrema problems you should check out some of the examples and/or practice problems there.

All we really need to do here is determine the absolute maximum of the profit function and the value of x that will give the absolute maximum.

Here is the derivative of the profit function and the critical point(s) since we'll need those for this problem.

$$P'(x) = -360,000 + 1500x - x^2 = -(x-1200)(x-300) = 0 \qquad \Rightarrow \qquad x = 300,\; x = 1200$$

Step 2
From the problem statement we can see that we only want critical points that are in the interval $[0,1500]$. As we can see both of the critical points from the above step are in this interval and so we'll need both of them.

Step 3
The next step is to evaluate the profit function at the critical points from the second step and at the end points of the given interval. Here are those function evaluations.

$$P(0) = 30,000,000 \qquad P(300) = -19,500,000$$
$$P(1200) = 102,000,000 \qquad P(1500) = 52,500,000$$

Step 4
From these evaluations we can see that they will need to sell 1200 widgets to maximize the profits.

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2. A management company is going to build a new apartment complex. They know that if the complex contains x apartments the maintenance costs for the building, landscaping etc. will be,

$$C(x) = 4000 + 14x - 0.04x^2$$

The land they have purchased can hold a complex of at most 500 apartments. How many apartments should the complex have in order to minimize the maintenance costs?

Step 1
Because these are essentially the same type of problems that we did in the Absolute Extrema section we will not be doing a lot of explanation to the steps here. If you need some practice on absolute extrema problems you should check out some of the examples and/or practice problems there.

All we really need to do here is determine the absolute minimum of the maintenance function and the value of x that will give the absolute minimum.

Here is the derivative of the maintenance function and the critical point(s) since we'll need those for this problem.

$$C'(x) = 14 - 0.08x = \qquad \Rightarrow \qquad x = 175$$

Step 2
From the problem statement we can see that we only want critical points that are in the interval $\left[0,500\right]$. As we can see both of the critical points from the above step are in this interval and so we'll need both of them.

Step 3
The next step is to evaluate the maintenance function at the critical points from the second step and at the end points of the given interval. Here are those function evaluations.

$$C\left(0\right)=4000 \qquad C\left(175\right)=5225 \qquad C\left(500\right)=1000$$

Step 4
From these evaluations we can see that the complex should have 500 apartments to minimize the maintenance costs.

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3. The production costs, in dollars, per day of producing x widgets is given by,

$$C\left(x\right)=1750+6x-0.04x^2+0.0003x^3$$

What is the marginal cost when $x=175$ and $x=300$? What do your answers tell you about the production costs?

Step 1
From the notes in this section we know that the marginal cost is simply the derivative of the cost function so let's start with that.

$$C'\left(x\right)=6-0.08x+0.0009x^2$$

Step 2
The marginal costs for each value of x is then,

$$\boxed{C'\left(125\right)=10.0625 \qquad C'\left(300\right)=63}$$

Step 3
From these computations we can see that is will cost approximately \$10.06 to produce the 126th widget and approximately \$63 to produce the 301st widget.

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4. The production costs, in dollars, per month of producing *x* widgets is given by,

$$C(x) = 200 + 0.5x + \frac{10000}{x}$$

What is the marginal cost when $x = 200$ and $x = 500$? What do your answers tell you about the production costs?

Step 1
From the notes in this section we know that the marginal cost is simply the derivative of the cost function so let's start with that.

$$C'(x) = 0.5 - \frac{10000}{x^2}$$

Step 2
The marginal costs for each value of *x* is then,

$$\boxed{C'(200) = 0.25 \qquad\qquad C'(500) = 0.46}$$

Step 3
From these computations we can see that is will cost approximately 25 cents to produce the 201^{st} widget and approximately 46 cents to produce the 501^{st} widget.

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5. The production costs, in dollars, per week of producing *x* widgets is given by,

$$C(x) = 4000 - 32x + 0.08x^2 + 0.00006x^3$$

and the demand function for the widgets is given by,

$$p(x) = 250 + 0.02x - 0.001x^2$$

What is the marginal cost, marginal revenue and marginal profit when $x = 200$ and $x = 400$? What do these numbers tell you about the cost, revenue and profit?

Step 1
First we need to get the revenue and profit functions. From the notes for this section we know that these functions are,

$$\text{Revenue : } R(x) = x\,p(x) = 250x + 0.02x^2 - 0.001x^3$$
$$\text{Profit : } P(x) = R(x) - C(x) = -4000 + 282x - 0.06x^2 - 0.00106x^3$$

Step 2
From the notes in this section we know that the marginal cost, marginal revenue and marginal profit functions are simply the derivative of the cost, revenue and profit functions so let's start with those.

$$C'(x) = -32 + 0.16x + 0.00018x^2$$
$$R'(x) = 250 + 0.04x - 0.003x^2$$
$$P'(x) = 282 - 0.12x - 0.00318x^2$$

Step 3
The marginal cost, marginal revenue and marginal profit for each value of x is then,

$C'(200) = 7.2$	$R'(200) = 138$	$P'(200) = 130.8$
$C'(400) = 60.8$	$R'(400) = -214$	$P'(400) = -274.8$

Step 4
From these computations we can see that producing the 201st widget will cost approximately \$7.2 and will add approximately \$138 in revenue and \$130.8 in profit.

Likewise, producing the 401st widget will cost approximately \$60.8 and will see a decrease of approximately \$214 in revenue and a decrease of \$274.8 in profit.