# Mr Honner

Math Appreciation

## Calculus Gave Me a Speeding Ticket

9 April 2013, 9:00 am


Years ago, one sunny Sunday afternoon, I was driving home from visiting friends at college and received a speeding ticket. I didn't realize it at the time, but calculus played an important role in my citation.

You see, this was no ordinary speeding ticket, the kind where a police officer paces the offender or uses radar to measure a vehicle's speed. My speed was calculated from an airplane high above the road. And the Mean Value Theorem clinched the case.

Aerial speed enforcement works like this: large marks painted on the road divide the highway into quarter-mile intervals. A pilot flying overhead uses a stopwatch to time a suspected speeder from one mark to the next. Say the pilot records a time of 12 seconds; a simple calculation converts one quarter mile per 12 seconds into 75 miles per hour; this information, the *average speed on this interval*, is radioed to the police on the ground who then stop and ticket the driver.

What I didn't realize at the time was how crucial calculus is in all of this.

A fundamental theorem of calculus, the *Mean Value Theorem* (MVT), relates the *average rate of change* of a function with the *instantaneous rate of change* of the function. Suppose we have some function of time, $f(t)$, and suppose that we know the value of this function at two times, say $f(t_1)$ and $f(t_2)$. The *average rate of change* of $f(t)$ between $t_1$ and $t_2$ is

$$f_{avg} = \frac{f(t_2)-f(t_1)}{t_2-t_1}$$

The MVT tells us that, as long as $f(t)$ is a *differentiable function*, then at some time between $t_1$ and $t_2$, say at *t* = *c*, the *instantaneous rate of change* of $f(t)$ must have been equal to the *average rate of change* of $f(t)$ from $t_1$ and $t_2$. That is,

$$f'(c) = \frac{f(t_2)-f(t_1)}{t_2-t_1}$$

where $f'(x)$ is the *derivative* of $f(x)$, the instantaneous rate of change of $f(x)$.

What does this have to do with my speeding ticket? Well, as I'm moving along the highway in my car, the pilot records two values of my *position* function, $x(t)$, at two different times, $t_1$ and $t_2$. The pilot then computes my *average speed*

$$x_{avg} = \frac{x(t_2)-x(t_1)}{t_2-t_1}$$

Here's where calculus comes in. The Mean Value Theorem says that, at some point between those two times my instantaneous speed must have been equal to my average speed. If my average speed was above the legal limit, then at some time between $t_1$ and $t_2$, my instantaneous speed must have been above the limit, and at that moment, I was guilty of speeding.

I wonder if it would have helped to argue that my position function wasn't differentiable!

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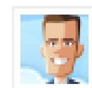
## 7 Comments

1. *Dan Meyer* says:
   April 10, 2013 at 9:58 pm

   Fun application, Patrick. I'm trying to figure out where I've seen these, though:

   "large marks painted on the road divide the highway into quarter-mile intervals"

   I can't see the trees for the forest, apparently. Any help?

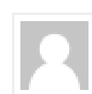
   - *MrHonner* says:
     April 10, 2013 at 10:17 pm

     I'm not sure this particular system is still in use, but at the time of the story (many years ago), white 'tick' marks were painted on the surface of the road along particular stretches of highway. The 'ticks' looked just like white lane divider marks, but they were drawn perpendicular to the flow of traffic along the side of the road. We measured the intervals to be around 1/4 mile, but who knows how accurate we were? We were usually driving too fast.

     Here's an image of what looks like a similar approach:
     http://www.waze.com/wiki/index.php/Image:UK_Cams_Gatso_Aerial.jpg

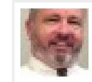
2. *Darius Mendez* says:
   May 3, 2013 at 11:53 pm

   it takes your average… if you start by speeding by 10 miles per hour and end at only 5 mph over the limit then it has you as speeding at 7.5 mph over the limit. This is only true if it measures you at the start and at the end only. if it measures you for the entire distance the average speed is not as easily calculated.

3. *Gene Chase* says:

May 28, 2013 at 11:56 am



When I was in NY State, the rule was that speeding was permitted for up to 1/4 mile while passing. You could have said, “I was passing.”

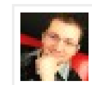
For that matter, if you were going 1,000 mph for 1 second, but 10 mph for your whole trip, you’re more likely to be arrested for hazardous driving than for speeding.

- *MrHonner* says:
  May 28, 2013 at 8:04 pm

  Unfortunately I wasn’t in NY at the time, Gene. But I’ll keep the “passing” defense in mind!

4. *Matt* says:
September 17, 2013 at 5:04 pm



I just received one of these good ole tickets. Wouldn’t the altitude play apart in the calculation that you have above and does anybody know if they will release information to me on how exactly they caught me? Like are the stop watches triggered by a human or device?, Are the stop watches calibrated and have certificates to prove that they are?, What was the altitude?, What was the current speed of the plane?, How many markers do they have and what is the distance between them? How many seconds did they record for my vehicle?

I just want to know so that I can calculate the speed by hand and prove that the system that they are using is so full of bugs and errors that it’s not even relevant to use in court. LOL Ok done with my rant, but I will seek answers and appear in court just to ask the pilot these same questions and prove that with a slight change in the variables my speed also changes.

- *MrHonner* says:
  September 18, 2013 at 7:07 am

  These are all good questions, Matt. I think there are a lot of little things taht could add up to a faulty reading: human error with the stopwatch and judging the start/finish; perspective error judging the start/finish; inaccurate measurement of the road; etc.

  Good luck! I’ll be shocked if the pilot shows up in court.