PHOTOGRAPH BY LEAH P. McCOY; ALL RIGHTS RESERVED

PROBABILITY GAMES from Diverse Cultures

LEAH P. McCOY, STEFANIE BUCKNER, AND JESSICA MUNLEY

TO MAKE MATHEMATICS RELEVANT AND meaningful for all students, it is important that we embrace a wide variety of real-world applications. Diverse cultures provide rich and interesting contexts in which students can experience and explore mathematics. One of the five Process Standards is Connections (NCTM 2000). The probability activities discussed here help students make connections across mathematics concepts through games from diverse cultures.

LEAH McCOY, *mccoy@wfu.edu, teaches mathematics education at Wake Forest University in Winston-Salem, North Carolina. She is interested in equity, technology, and teaching mathematics for conceptual understanding.* STEFANIE BUCKNER, *sbuckner1@wcpss.net, and* JESSICA MUNLEY, *jmunley@wcpss.net, teach mathematics at Wake Forest-Rolesville High School in Wake Forest, North Carolina. They are interested in problem solving and real-world connections.*

Each lesson followed a common format. First, students learned about the game, including the history and background, as well as instructions for playing. Second, the teacher demonstrated the game to the class. Then students were placed in small groups of two to four, given appropriate game materials, and instructed to play. They were then introduced to a probability concept related to the game, either as an integral part of the strategy or as an experiment where data were collected as the game was played. Students collected and analyzed the data, and reported their results on the group worksheets as both short answers and longer explanations.

The lessons were designed and tested in prealgebra classes in a rural public middle school. Materials were either teacher-made or inexpensively purchased. We used the games as independent lessons, but a "game fair," where small groups of students rotate through the games in different learning centers, could also be designed.

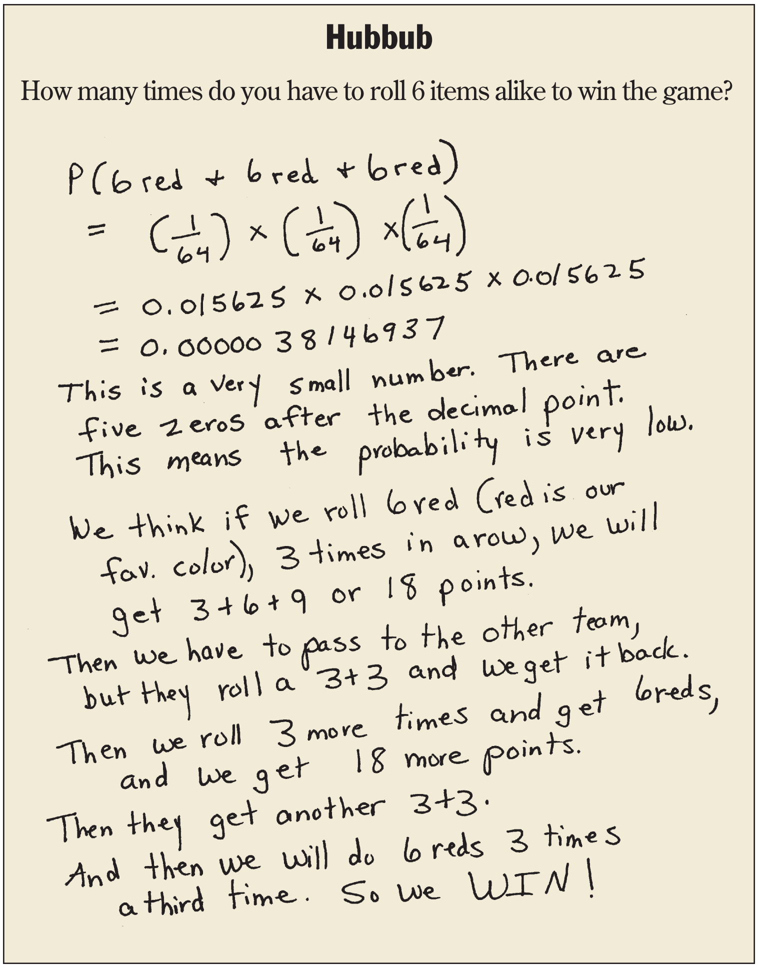
Copyright © 2007 The National Council of Teachers of Mathematics, Inc. www.nctm.org. All rights reserved.
This material may not be copied or distributed electronically or in any other format without written permission from NCTM.

Game 1: Hubbub (Native American)

BOWL AND DICE (WA'LADE HAMA'GAN), A SIMPLE game of chance, is one of many similar games played by Native Americans (Culin 1992). The Penobscot Nation in New England called it *hubbub* because of the chanting that accompanied the two-sided "dice" being tossed in a bowl or basket. The dice used were carved from bone or antler or were animal teeth, peach pits, or small stones that had been engraved, burned, polished, or painted to distinguish the sides. For dice, we used lima beans that had been spray painted on one side, and plastic bowls were used to hold the beans. Players flip one of the beans like a coin to determine who goes first, then that player tosses 6 bean dice in the bowl. The following scoring rules are used:

- Six beans alike: 3 points and take another turn
- A second six alike: 6 points and take another turn
- A third six alike: 9 points and pass the turn
- Five alike: 2 points and take another turn
- A second five alike: 4 points and take another turn
- A third five alike: 6 points and pass the turn
- Less than five alike or a nonrepeat on the second or third toss: No points and pass the turn
- Continue playing until one player reaches 50 points

This game is used as a context for defining simple probability. See the sample space on the "Hubbub Game Exercises" worksheet and the questions given to students. First, students found the probability of 1 bean landing white or red side up as the (number of favorable outcomes)/(total outcomes) = 1/2. Students were then asked to construct a tree diagram of the sample space. If they wanted 6 beans to land on the red side, this was one outcome of 64 in the sample space. After obtaining these fractions, students either used the fraction form (1/64) or used a calculator to convert it to a decimal (0.015625) and discussed how small the probability was. In both cases, the class concluded that these were small amounts, and this was a small probability. They also discussed the meaning of "and" in P(all red) *and* P(all red, again). This gave them hands-on experience with compound probability and even smaller fractions ($1/64 \times 1/64 = 1/4096$). Students grasped the concept of probability and compound probability and were able to explain that the small probability of one toss of all red became much smaller when considering the probability of one all red and then another all red. With some encouragement, most groups were able to answer the final open-ended questions that required them to apply the concepts and communicate their mathematical thinking. See **figure 1** for one group's written report.


Hubbub

How many times do you have to roll 6 items alike to win the game?

P(6red + 6red + 6red)
= (1/64) × (1/64) × (1/64)
= 0.015625 × 0.015625 × 0.015625
= 0.0000038146937
This is a very small number. There are five zeros after the decimal point. This means the probability is very low.

We think if we roll 6red (red is our fav. color), 3 times in a row, we will get 3+6+9 or 18 points.
Then we have to pass to the other team, but they roll a 3+3 and we get it back.
Then we roll 3 more times and get 6reds, and we get 18 more points.
Then they get another 3+3.
And then we will do 6reds 3 times a third time. So we WIN!

Fig. 1 Student work from the hubbub game

PHOTOGRAPH BY LEAH P. McCOY; ALL RIGHTS RESERVED

Game 2: Mancala (African)

MANCALA (ALSO CALLED WARI) IS A COMMON game played throughout Africa and the Caribbean. There are many variations and names for this game, including ayo in Nigeria, omweso in Uganda, and kalaha in Egypt. Mancala was played before 1400 BC in ancient Egypt, and many historians believe that it is the oldest game in the world (Hanson 2004).

PHOTOGRAPH BY LEAH P. McCOY; ALL RIGHTS RESERVED

Fig. 2 The mancala game being played in class with an egg carton and beans

Mancala:

Describe whether starting first or second is related to whether you win the game.

In our experiment, the person who played first won the game 4 out of 5 times. We don't know why this is, but we think it might be because there are 8 pits on the board, and first is an odd number. And if you don't count the pit where you started, then there are an odd number of pits.

We don't think it matters who starts the game first. We played four times and the person starting won 2 times and the person going second won 2 times so it must be random.

When the class added all of the data together, it turned out that the person who started first also won the most. It did not matter if they were called player 1 or player 2.

Fig. 3 Examples of student work from mancala, describing winning and playing order

The Arabic word *mancala* means "to transfer." The game is played by transferring small playing pieces, usually stones or seeds, from one pit to another. In its simplest version, the game board is a patch of ground, and the pits are depressions dug into the earth. Mancala boards were usually carved of stone, wood, or ivory, and were sometimes elaborate and artistic. Many are displayed in art museums across the world.

In our class, the boards were half-dozen-sized egg cartons (with three pits on each side), separate bowls were placed at each end, and the playing pieces were beans. (See **fig. 2**.) Traditional boards have six or eight pits on each side. The directions for the game are listed below.

- At the beginning, each pit contains four beans, which represent seeds. Players alternate turns. The first player chooses one pit from which to "sow" the "seeds."
- Each bean in the pit is then placed, one at a time, into successive pits, moving counterclockwise around the board. Beans placed in the *kalaha*, the "pit" at the end of the board to the player's right, are counted as points for that player. Beans are not sown in the opponent's kalaha.
- If the last bean in a play is placed in the player's own kalaha, he or she gets another turn.
- If the last bean is placed in an empty pit on the player's side of the board, then he or she captures the beans in the opponent's pit that is opposite the ending pit. All captured beans, as well as the capturing piece, are placed in the player's kalaha.
- The game ends when all the pits on one side of the board are empty.
- The winner is the player with the most beans in the kalaha.

Most students had never played this game but caught on quickly. Each team of two played several times and collected data regarding which player started first and which player won. Students were asked to conjecture whether there was a relationship between starting first and winning the game. (See **fig. 3**.)

The data were then combined for the entire class and used to find the experimental probabilities of whether the starting player won or lost. Finally, students were asked to write a paragraph describing the relationship of starting and winning in their experiment, giving them an opportunity to answer a broad open-ended question and focus on reasoning skills. The experimental probabilities will vary, but examining the class data should lead the students to conclude that winning the game is more about strategy than starting order. See the worksheet titled "Mancala Exercises."

This mancala activity explored basic experimental probability. Further explorations could require students to develop strategies and planning about distributing beans and choosing the starting pit. Mancala has been compared with the game of chess because of its extensive and intricate strategy possibilities.

Game 3: Toma Todo (Mexican)

TOMA TODO, OR PIRINOLA, IS A GAME OF CHANCE from Mexico. Children as well as adults play toma todo with a *pirinola,* which is a six-sided top usually found in Hispanic gift shops or variety stores (Krause 2000) (see the item on the right in **fig. 4**). Each of six possible outcomes on the pirinola is equally likely to occur. To win, each player must acquire all the beans from the other players.

- To begin the game, each player has 10 beans and places 1 bean in a center pile. Turns are taken. A turn consists of spinning the pirinola and following the directions on the side that is faceup after the spin.
- The sides of the pirinola contain this information: pon 1 (put in 1 bean), pon 2 (put in 2), toma todo (take all), toma 1 (take 1), toma 2 (take 2), and todos ponen (each player puts 1 in the pile).
- If the toma todo side lands faceup, then to continue the game, each player must place 1 bean in a center pile again. Play continues until one player has all the beans.

A probability exercise for toma todo

1. It appears that the pirinola is fair in that each of the six sides is equally likely to turn up. What is the theoretical probability of each side (outcome) if this is true?
2. When playing the game, keep a tally of the number of times each side turns up. Using these data, calculate the experimental probability of landing up for each side, and then sum the probabilities of all players.
3. Calculate the experimental probability of landing up for each side for the winner. Did the winner have a higher probability of some outcomes than other players? Explain the results. Compare your conclusions with other groups in the class.

Game 4: Dreidel (Jewish)

ONE OF THE BEST-KNOWN SYMBOLS OF HANUKkah is the dreidel, which is a four-sided top in-

PHOTOGRAPH BY LEAH P. McCOY; ALL RIGHTS RESERVED

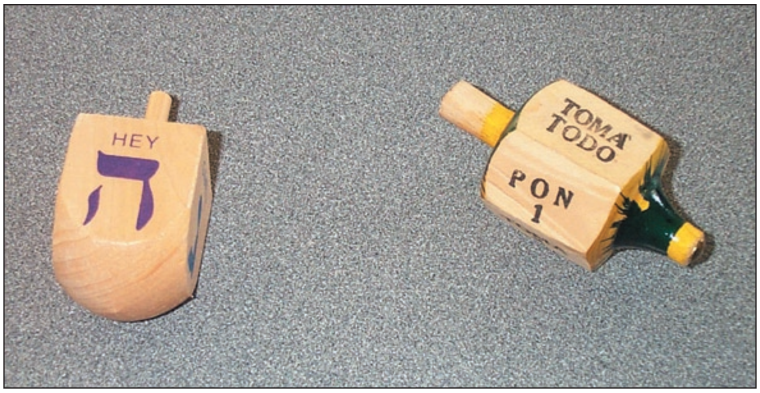


Fig. 4 Two top-like spinners: at left is a dreidel; at right, a pirinola

scribed with a Hebrew letter on each side. (See the item on the left in **fig. 4**.) The letters stand for "Nes gadol hayah sham," which means "A great miracle happened there." (In Israel, the dreidel reads "Nes gadol hayah poh," which means "A great miracle happened here.") The miracle refers to the small flask of oil that burned for eight days (Krause 2000). The dreidel has the following symbols on its four faces and sometimes contain the words, as well.

נ	ג	ה	ש
Nun	Gimel	Hey	Shin

Nun stands for "nothing"; if the dreidel lands on this side, you do nothing. *Gimel* stands for "all" and means "take everything in the middle." *Hey* stands for "half." The player takes half of what is in the middle plus 1 if there is an odd number of objects. *Shin* stands for "put in," meaning "put two objects into the middle." Dreidels can be purchased at holiday stores and toy stores, and inexpensive plastic or wooden ones can be found. Players use pennies, nuts, raisins, or chocolate coins (gelt) as tokens or chips. To begin the game, each player places an agreed-on number of tokens in the center of the table. Each player spins the dreidel in turn. When it stops, the letter facing up decides the action. The game is over when one player has won all the tokens or after playing a designated number of rounds.

A probability exercise for the dreidel

1. It appears that the dreidel is fair in that each of the four faces is equally likely to turn up. What is the theoretical probability of each side (outcome) if this is true?
2. When playing the game, keep a tally of the number of times each side turns up. Using these data,

calculate the experimental probability of landing up for each side, then find the sum of all the players' probabilities.

3. Are the theoretical probability and the experimental probabilities for each face the same? Why or why not?
4. Consider the player who won the game (who had the most tokens at the end). Did he or she have different experimental probabilities than other players? Explain the results. Compare with other groups in the class.

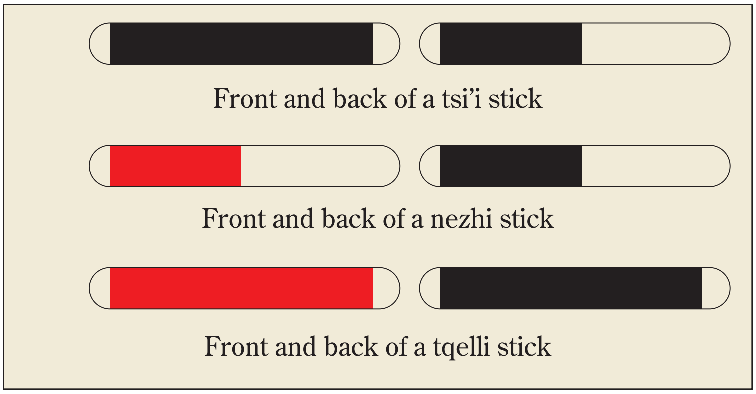


Fig. 5 Colored sticks are used to play and score ashbii.

Game 5: Ashbii (Native American)

ASHBII (ASH BEEN) WAS A GAME PLAYED BY Navajo women and children while they sat under a buffalo hide that had been staked for drying (Culin 1992). The game is played with three painted sticks, representing dice, which are tossed in a basket. The sticks can be made by spray painting craft sticks. The first stick is called the *tsi'i* (zeen) and is black on one side and one-half black on the other. The second stick is called *nezhi* (nezshi) and is one-half red on one side and one-half black on the other. The third stick is called *tqelli* (zelli) and is painted red on one side and black on the other. (See **fig. 5** for a diagram of the colored sticks.) All players sit on the floor and alternate throwing the sticks. The three sticks are tossed upward, ideally against a blanket stretched overhead to resemble a staked buffalo hide, and the sticks that land in the basket are scored. Any stick not landing in the basket is ignored. "Crossing" refers to sticks that intersect and results in higher scores. Red crossing red is 5 points. Red crossing black is 3 points. Any red is 2 points, and any black is 1 point. Only the highest scoring combination that occurs on any one toss will count. The winner is the

Tsi'i (Ts)	Nezhi (N)	Tqelli (Tq)		Sides Up Ts-N-Tq	Points Ts-N		Points Ts-Tq		Points N-Tq	
					Cross	Not	Cross	Not	Cross	Not
All Black	Half Red	All Red	=	**AB-HR-AR**	3	2	3	2	5	2
		All Black	=	**AB-HR-AB**	3	2	1	1	3	2
	Half Black	All Red	=	**AB-HB-AR**	1	1	3	2	3	2
		All Black	=	**AB-HB-AB**	1	1	1	1	1	1
Half Black	Half Red	All Red	=	**HB-HR-AR**	3	2	3	2	5	2
		All Black	=	**HB-HR-AB**	3	2	1	1	3	2
	Half Black	All Red	=	**HB-HB-AR**	1	1	3	2	3	2
		All Black	=	**HB-HB-AB**	1	1	1	1	1	1

"Crossing" is defined as intersecting sticks. In each case, only the highest scoring combination that occurs on any one toss will count. Thus, the player will get one of the scores above for each toss. The winner is the player who first scores 25 points.

Fig. 6 Possible outcomes and scoring for ashbii

player who first scores 25 points. (See **fig. 6** for the possible outcomes and scoring.)

A probability exercise for ashbii

1. How many different ways can the sticks fall (outcomes)? Make a table of all of the possible outcomes. Do not forget the crossed tosses and those that do not earn points.
2. What is the probability of each outcome?
3. Each toss may earn a score of 1, 2, 3, or 5 points. What is the probability of each score on one toss?
4. Is this a fair way to score the game? Is there a relationship between the probability of each outcome and its score?

Game 6: Lu-Lu (Hawaiian)

EARLY HAWAIIANS PLAYED LU-LU WITH DISKS OF volcanic stone about 2.5 cm in diameter (Krause 2000). These disks served as stone dice called *u-lu*. The word *lu-lu* means "to shake." The dice can be made from clay, shell, or wood. (Glass stones used by florists are inexpensive and can be painted on one side only in the design shown in **fig. 7**.)

The basic version of lu-lu consists of players taking turns tossing all four stones. The dots on the pieces that fall faceup are counted. A player shakes the stones in both hands and tosses them. The first player to reach 50 wins the game. This variation of dice is a good way to give students additional practice constructing sample spaces and calculating basic probabilities. There are sixteen possible outcomes in this basic version of the game.

A probability exercise for basic lu-lu

1. What are all the possible outcomes in one toss? Make a diagram.
2. What scores are possible for a turn?
3. How many different ways can each score be obtained?
4. What is the probability of each score?

In the challenge version of this game, a turn consists of two tosses. On the first toss, if all 4 stones fall faceup, the player scores 10, then tosses all dice again. If all 4 do not fall faceup on the first toss, the dots of those faceup are scored and only the facedown pieces are tossed a second time. The dots showing on the second toss are added to those from the first toss. The winner is the player who first reaches a score of 100. This problem involves more possibilities. The tree diagram begins with sixteen basic possibilities, then branches to consider all the possibilities of those stones that landed facedown on the first toss.

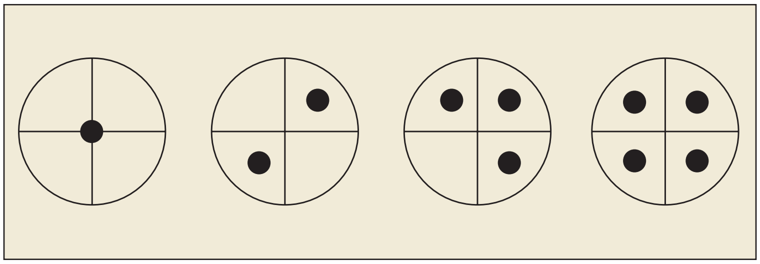


Fig. 7 The designs painted on stone dice for playing lu-lu

A probability exercise for the challenge version of lu-lu

1. What are all the possible outcomes in two tosses? Consider the retossed dice. Order does not count. Make a diagram.
2. What scores are possible for a turn?
3. How many different ways can each score be obtained?
4. What is the probability of each score?

Conclusions

THESE GAMES WERE USED SUCCESSFULLY WITH middle schoolers for a number of reasons. First, the games provided a rich and interesting context for applying important probability concepts. The activities would be equally appropriate as an introductory activity when beginning a unit on probability or as a review or enrichment at the end. Second, these games connected students to diverse cultures, which is sometimes difficult to do in mathematics class. Third, these lessons used sound pedagogy. Students were actively involved in the activities and worked both individually and in groups to complete the mathematics exercises and communicate their understandings. It is recommended that these and similar games be integrated throughout the mathematics curriculum.

Bibliography

Culin, Stewart. *Games of the North American Indians. Volume 1: Games of Chance*. Lincoln, NE: University of Nebraska Press, 1992.

Hankes, Judith Elaine, and Gerald R. Fast, eds. *Changing the Faces of Mathematics: Perspectives on Indigenous People of North America*. Reston, VA: National Council of Teachers of Mathematics, 2002.

Hanson, Sue. *All about Mancala: Its History and How to Play*. Mahomet, IL: Happy Viking Crafts, 2004.

Krause, Marina C. *Multicultural Mathematics Materials*. 2nd ed. Reston, VA: National Council of Teachers of Mathematics, 2000.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000. ☞

Answer Key to the Worksheets and Probability Exercises

"Hubbub Game Exercises" Worksheet
1. 8
2. 3
3. 3/8
4. 1/8
5. 64
6. 6/64, or 3/32
7. 9/1024
8. 27/32768
9. 20/64, or 5/16
10. 1/64
11. 1/64
12. 1/4096
13. 1/262144; 18
14. 3

"Mancala Exercises" Worksheet
Answers will vary.

Toma Todo Probability Exercise (p. 397)
1. 1/6.
2.–3. Answers will vary.

Dreidel Probability Exercise (pp. 397–98)
1. 25/100
2.–4. A answers will vary.

Ashbii Probability Exercise (p. 399)
1. 48 (see **fig. 6**)
2. 1/48
3. 5 is 2/48, 3 is 12/48, 2 is 14/48, 1 is 20/48
4. Answers will vary.

(Basic) Lu-lu Probability Exercise (p. 399)
1. There are 16 possible outcomes; each of the four stones may be up or down: 4-3-2-1; 4-3-2-0; 4-3-0-1; 4-3-0-0; 4-0-2-1; 4-0-2-0; 4-0-0-1; 4-0-0-0; 0-3-2-1; 0-3-2-0; 0-3-0-1; 0-3-0-0; 0-0-2-1; 0-0-2-0; 0-0-0-1; 0-0-0-0.
2. Eleven scores are possible: 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0.
3. 10, 9, 8, 2, 1, 0 are scored once each, and 7, 6, 5, 4, 3 are scored twice each.
4. 10 is 1/16, 9 is 1/16, 8 is 1/16, 7 is 2/16, 6 is 2/16, 5 is 2/16, 4 is 2/16, 3 is 2/16, 2 is 1/16, 1 is 1/16, 0 is 1/16.

(Challenge) Lu-lu Probability Exercise (p. 399)
1. 96 possible outcomes

Toss 1	Toss 2
4-3-2-1	(4-3-2-1, 4-3-2-0, 4-3-0-1, 4-3-0-0, 4-0-2-1, 4-0-2-0, 4-0-0-1, 4-0-0-0, 0-3-2-1, 0-3-2-0, 0-3-0-1, 0-3-0-0, 0-0-2-1, 0-0-2-0, 0-0-0-1, 0-0-0-0)
4-3-2-0	(1, 0)
4-3-0-1	(2, 0)
4-3-0-0	(2-1, 2-0, 0-1, 0-0)
4-0-2-1	(3, 0)
4-0-2-0	(3-1, 3-0, 0-1, 0-0)
4-0-0-1	(3-2, 3-0, 0-2, 0-0)
4-0-0-0	(3-2-1, 3-2-0, 3-0-1, 3-0-0, 0-2-1, 0-2-0, 0-0-1, 0-0-0)
0-3-2-1	(4, 0)
0-3-2-0	(4-1, 4-0, 0-1, 0-0)
0-3-0-1	(4-2, 4-0, 0-2, 0-0)
0-3-0-0	(4-2-1, 4-2-0, 4-0-1, 4-0-0, 0-2-1, 0-2-0, 0-0-1, 0-0-0)
0-0-2-1	(4-3, 4-0, 0-3, 0-0)
0-0-2-0	(4-3-1, 4-3-0, 4-0-1, 4-0-0, 0-3-1, 0-3-0, 0-0-1, 0-0-0)
0-0-0-1	(4-3-2, 4-3-0, 4-0-2, 4-0-0, 0-3-2, 0-3-0, 0-0-2, 0-0-0)
0-0-0-0	(4-3-2-1, 4-3-2-0, 4-3-0-1, 4-3-0-0, 4-0-2-1, 4-0-2-0, 4-0-0-1, 4-0-0-0, 0-3-2-1, 0-3-2-0, 0-3-0-1, 0-3-0-0, 0-0-2-1, 0-0-2-0, 0-0-0-1, 0-0-0-0)

2. 21 scores (0–20) are possible.
3. 20, 19, 18, 12, 11, 0 are scored once each; 17, 16, 15, 14, 13, 2, 1 are scored twice each; 4, 3, six times each; 9, 8, 5, eight times each; 7, 6, twelve times each; 10, sixteen times
4. 20, 19, 18, 12, 11, 0 are 1/96; 17, 16, 15, 14, 13, 2, 1 are 2/96; 4, 3 are 6/96; 9, 8, 5 are 8/96; 7, 6 are 12/96; 10 is 16/96 □

Hubbub Game Exercises

NAME ______________________________

Each bean has two different sides. Thus, there are two possible ways that this bean could land. If these outcomes are equally likely, the probability (or chance) that this bean lands red side up is the number of ways that red could appear divided by the total number of ways the bean can land. In probability notation and as a number, $P(\text{red}) = 1/2$.

Construct a tree diagram of the sample space for tossing *three* two-sided dice.

1. How many different outcomes are possible? __________

2. How many ways can you get 2 white beans and 1 red bean? ________

Remember, the probability is the number of favorable outcomes over the total number of possible outcomes. Order does not matter.

3. What is *P*(2 white beans and 1 red bean)?_________

4. What is *P*(3 red beans)?__________

Now construct a tree diagram of the sample space for tossing *six* two-sided dice.

5. How many outcomes are possible? __________

6. *P*(5 red beans and 1 white bean) __________

7. *P*(rolling 5 red beans and 1 white bean, two times in a row) __________
(Hint: "First roll *and* then second roll" requires multiplication of the probabilities.)

8. *P*(rolling 5 red beans and 1 white bean, three times in a row) __________

9. *P*(3 red beans and 3 white beans) __________

10. *P*(6 red beans) __________

11. *P*(6 white beans) __________

12. *P*(all red) and *P*(all red again) __________

13. Find the probability of rolling all 6 red beans, three times in a row. __________
What would your score be? __________

14. Assuming that you have beaten the odds and rolled all 6 of the same color (either all red or all white) each time, how many times do you have to do this to win the game? Explain.

Mancala Exercises

NAME ______________________________

Quick Review

The *probability* of an event, or *P*(event), tells you how likely it is that something will occur. The *experimental probability* is based on data collected from repeated trials. You can find the experimental probability of an event by using this formula:

$$P(\text{event}) = \frac{\text{number of times an event occurs}}{\text{number of times the experiment is done}}$$

Play mancala at least five times and record your outcomes in the following table. After all groups have collected data, we will combine data for all groups and fill in the class totals together.

Event	Our Group	Class Totals
Player 1 begins, player 1 wins		
Player 1 begins, player 1 loses		
Player 2 begins, player 2 wins		
Player 2 begins, player 2 loses		

Using the combined class data, answer the following questions. Show your work.

1. What is the probability that if player 1 begins, player 1 will win? _________

2. What is the probability that if player 1 begins, player 1 will lose? _________

3. What is the probability that if player 2 begins, player 2 will win? _________

4. What is the probability that if player 2 begins, player 2 will lose? _________

5. What is the probability that player 1 will win, regardless of the starting position? _________

6. What is the probability that player 2 will win, regardless of the starting position? _________

7. Write a paragraph explaining whether starting first or starting second is related to winning the game.