

Math 499 □ History of Mathematics □ Spring 2014

Practice Final April 22, 2014

1. CALCULUS

1.1. Question 1.

1. (4 points) State and prove the product rule.
2. (2 points) Find the derivative function $f'(x)$ for $f(x) = \frac{x^2}{x \cos(x)}$.
3. (4 points) On the Moscow papyrus dating back to 1850 BC, the general formula $V = h(a^2 + ab + b^2)/3$ for a truncated pyramid with base length a , roof length b and height h appeared. Assume $h(t) = 1 + \sin(t)$, $a(t) = 1 + t$, $b(t) = 1 - 2t$. Does the volume of the truncated pyramid grow or decrease at rst?

1.2. Question 2. The Brachystochrone Problem.

1. (2 points) What is the Brachystochrone Problem and who stated the problem? Who are the mathematician that submitted a solution?
2. (4 points) What is the solution? Explain in details and draw a picture.
3. (4 points) Which principle did Johan Bernoulli used in his proof and how?

2. KÖNISBERG BRIDGES.

2.1. Question 1. (2 points) Explain the Problem of the Könisberg bridges. Draw a picture to illustrate the question.

2.2. Question 2. (2 points) Who is L.Euler and what is his relation to the Problem of the Könisberg bridges? Write one paragraph about him.

2.3. Question 3. (8 points)

1. What was the first step taken by the mathematician to analyze the problem? (1 point)

2. Define what is a graph, a vertex, an edge. (2 points)
3. How did Euler found the solution? Explain in details. (2 points)

3. GRAPH THEORY.

3.1. **Question 1.** Provide a complete definition of an Euler path and an Euler circuit (2 points).

1. Draw a graph that does not have an Euler graph. (1 points)
2. Draw a graph that does not have an Euler circuit. (1 points)
3. Can you find a graph with exactly one odd vertex? Justify your answer (1 point)
4. Explain the necessary condition to have an Euler Circuit.

3.2. **Question 2.** (2 points) Provide a graph with three odd vertices. Justify your answer.

3.3. **Question 3.** (4 points) Describe Fleury's algorithm and illustrate with an example. Define what a bridge is.

4. FOUR COLOR THEOREM

4.1. **Question 1.** (4 points) Explain clearly what is the four color theorem. Illustrate with a picture.

4.2. **Question 2.** (4 points) Explain the historical details in the proof of the four color theorem. Why did it play an important role in the world of mathematics?

4.3. **Question 3.** (4 points) What is the Euler Theorem used to demonstrate the six color theorem? Add a picture to illustrate the result.

5. MATHEMATICIANS

5.1. **Question 1.** (8 points) Choose a mathematician from the 18th-19th or 20th century and write two paragraphs about him/her. You need to provide the dates, some specific about the personality of the mathematician as well as the contribution to mathematics.