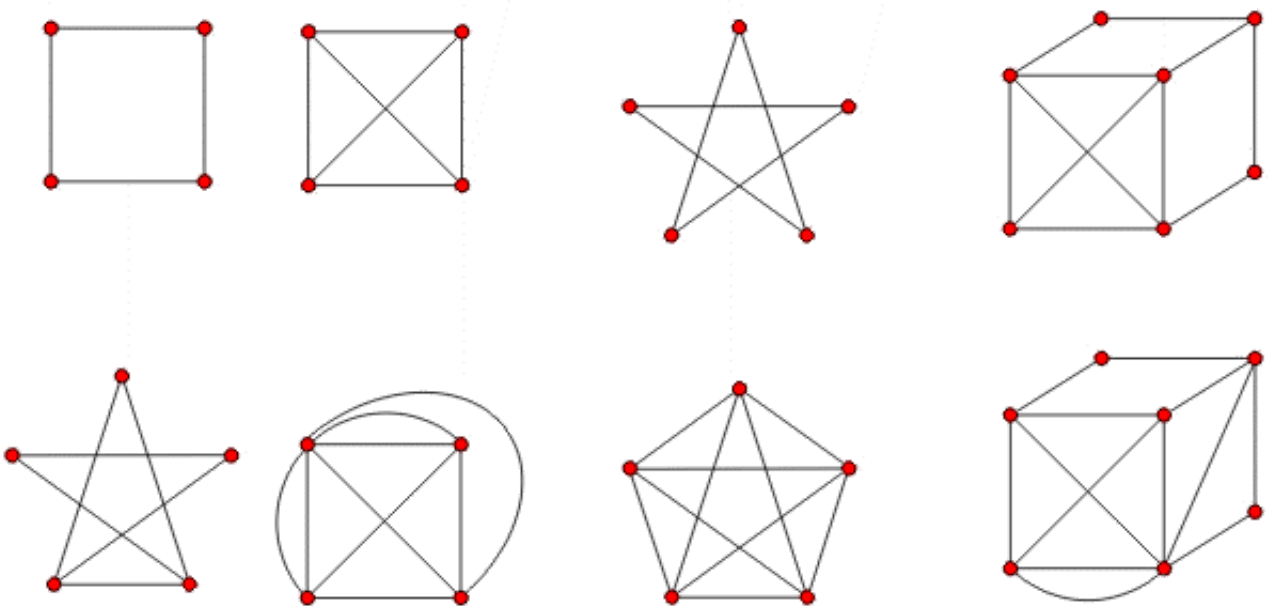
Walking Spiders

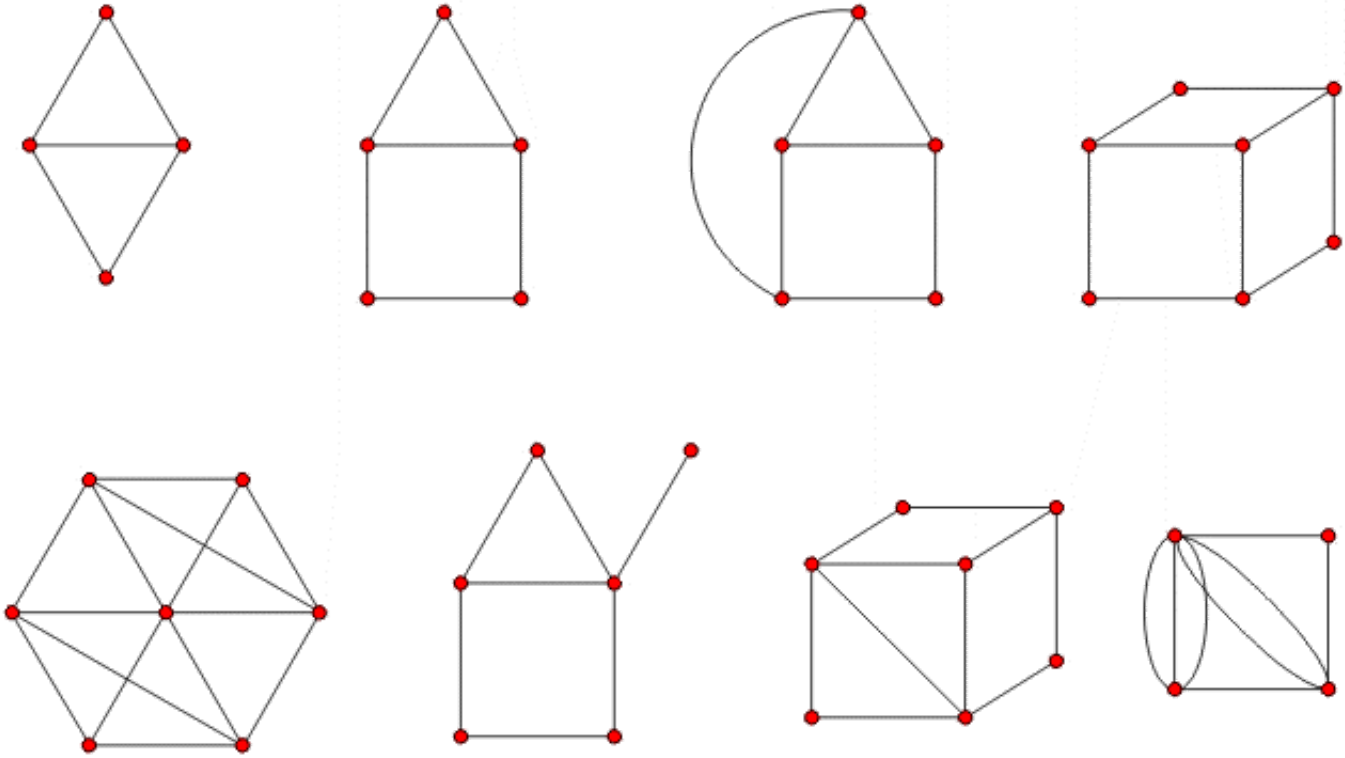
In this question we will address the following question: given a three dimensional solid, or a two dimensional graph, can a spider walk across the edges in such a way that it crosses each edge exactly once? This type of problem was considered first by the residents of Koenigsburg. There are seven bridges crossing the river that divides the city. People wondered if one could walk around the city crossing every bridge exactly once. While nobody had ever accomplished this, they did not know if it was impossible. Euler invented graph theory to solve this problem.

Problem 1. With each of the graphs below, try to find a path that crosses each edge exactly once and starts and ends at the same vertex. If you find such a path, indicate the path you found.



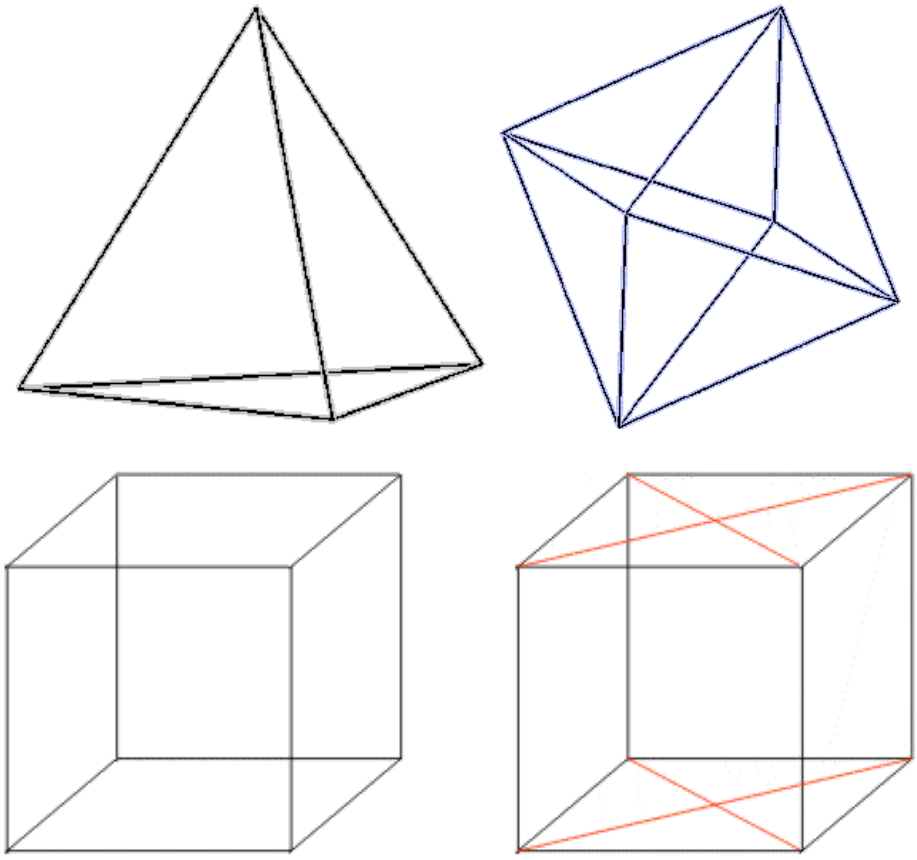
Problem 2. Based on the examples above, come up with a conjecture (an educated guess) about what it takes to be able to find a path that crosses every edge exactly once and starts and ends at the same vertex. To help you get the right idea, determine the number of edges connected to each vertex, and write these numbers next to the vertices. Try to find a pattern between these numbers and whether the graph has the appropriate path.

Problem 3. With each of the graphs below, try to find a path that crosses each edge exactly once. You need not start and end at the same vertex. In fact, no graph below allows such a path that starts and ends at the same vertex.



Problem 4. Based on the examples above, come up with a conjecture about what it takes to be able to find a path that crosses every edge exactly once. You should approach this in the same way as you did in Problem 2. Also look carefully at the start and end of your paths to be very specific about your guess.

Problem 5. Using your conjectures, determine whether or not a spider can find a path that crosses each edge of the following three dimensional solids exactly once. If it can be done, find such a path.



Problem 6. Take the following map of Koenigsburg, and draw a graph where each land mass is a vertex, and two masses are connected if there is a bridge connecting them. There are four land masses; two islands together with the land above and the land below the river. Determine whether or not a person can walk across each bridge exactly once.