Math 215 Precalculus Review Notes

Formulas
area of circle: $A = \pi r^2$
circumference of a circle: $C = 2\pi r$
volume of a sphere: $V = \frac{1}{3}\pi r^3$
surface of a right circular cylinder: $S = 2\pi r^2 + 2\pi rh$
volume of a right circular cylinder: $V = \pi r^2 h$
Pythagorean Theorem: In a right triangle with hypotenuse length c and legs with lengths b and a, then $a^2 + b^2 = c^2$.

Properties of the Exponential Function

1. $b^0 = 1$ and $b^1 = b$
2. $b^x b^y = b^{x+y}$
3. $(b^x)^y = b^{xy}$
4. $b^{-x} = \dfrac{1}{b^x}$
5. $\dfrac{b^x}{b^y} = b^{x-y}$

Properties of the Logarithm

The logarithm is the inverse of the exponential function. Thus,

$$\log_b(b^x) = \log_b(\exp_b x) = x \qquad \text{and} \qquad \exp_b(\log_b x) = b^{\log_b x} = x.$$

Special cases,

$$\log_b(b) = \log_b(b^1) = 1 \qquad \text{and} \qquad \log_b(1) = \log_b(b^0) = 0.$$

1. Property 1: $\log_b PQ = \log_b P + \log_b Q$
2. Property 2: $\log_b P^n = n \log_b P$
3. Property 3: $\log_b \frac{P}{Q} = \log_b P - \log_b Q$
4. Property 4: (Change of Base Formula) $\qquad \log_a x = \dfrac{\log_b x}{\log_b a}$

Trigonometry

Definitions

1. $\sin\theta = \dfrac{opposite}{hypotenuse}$
2. $\cos\theta = \dfrac{adjacent}{hypotenuse}$
3. $\tan\theta = \dfrac{opposite}{adjacent}$
4. $\csc\theta = \dfrac{1}{\sin\theta}$
5. $\sec\theta = \dfrac{1}{\cos\theta}$
6. $\cot\theta = \dfrac{1}{\tan\theta}$

Identities

1. $\sin^2\theta + \cos^2\theta = 1$
2. $\tan^2\theta + 1 = \sec^2\theta$
3. $1 + \cot^2\theta = \csc^2\theta$
4. $\cos(-\theta) = \cos\theta$
5. $\sin(-\theta) = -\sin\theta$
6. $\cos(\theta - \frac{\pi}{2}) = \sin\theta$
7. $\sin(\theta - \frac{\pi}{2}) = -\cos\theta$
8. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$
9. $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
10. $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$
11. $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$
12. $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
13. $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$
14. $\sin(2\theta) = 2\sin\theta\cos\theta$
15. $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
16. $\tan(2\theta) = \dfrac{2\tan\theta}{1 - \tan^2\theta}$
17. $\cos(\frac{\theta}{2}) = \sqrt{\dfrac{1 + \cos\theta}{2}}$
18. $\sin(\frac{\theta}{2}) = \sqrt{\dfrac{1 - \cos\theta}{2}}$