*INSTRUCTIONS:* Write legibly. Indicate your answer clearly. Revise and clean up solutions. Do not cross anything out. Rewrite the page, I will not grade problems where any part is crossed out. Make sure all steps of your solution are in order. If you write things in the margin I will ignore your solution. In other words, the standards on appearance will be high. Show all work; explain your answers. Answers with work not shown might be worth **zero** points.

There will be credit for good and complete solution, and esentially no partial credit.

| Problem | Worth | Score |
|---------|-------|-------|
| 1 | 1 | |
| 2 | 1 | |
| 3 | 1 | |
| 4 | 1 | |
| 5 | 1 | |
| 6 | 1 | |
| 7 | 1 | |
| 8 | 1 | |
| 9 | 1 | |
| 10 | 1 | |
| 11 | 1 | |
| 12 | 1 | |
| 13 | 1 | |
| 14 | 1 | |
| Total | 14 | |

(1)   1. Work out the following limits, or show that they do not exist:

(a) $\lim\limits_{x\to 0} \dfrac{\tan(3x)}{5x} =$

(b) $\lim\limits_{x\to 16} \dfrac{\sqrt[4]{x}-2}{x-16} =$

(c) $\lim\limits_{x\to 0} \dfrac{\sec^2 x - 1}{\tan x} =$

(d) $\lim\limits_{x\to 1} \dfrac{x^3 - 4x^2 + 8x - 5}{x^2 - 2x + 1} =$

(1) 2. Find the derivatives of each of the following functions! Do not simplify!

(a) $f(x) = \dfrac{x^3 - 2x + 1}{\cos x + 1}$

(b) $g(x) = x^2 \sin^3(2x)$

(c) $h(x) = \tan^3(7x^2 + 5)$

(d) $i(x) = \sqrt{3x^2 - \sin^2 x}$

(e) $j(x) = \dfrac{(x^2 + 1)\sec(2x)}{x^2 - 1}$

(1)   3. Work out the following integrals:

(a) $\displaystyle\int_{-1}^{1} t^3(1+t^4)^3\, dt$

(b) $\displaystyle\int_{0}^{1} \frac{5r}{(4+r^2)^2}\, dr$

(c) $\displaystyle\int_{-\pi/2}^{\pi/2} (2+\tan(t/2))\sec^2(t/2)\, dt$

(d) $\displaystyle\int_{0}^{\sqrt{7}} t(t^2+1)^{1/3}\, dt$

(e) $\displaystyle\int_{1}^{2} t(t-1)^{4/7}\, dt$

(1) 4. Consider the function $f(x) = 1/x^2$ on the interval $[1, 2]$. Work out Riemann sums using left endpoints, right endpoints and midpoints. Use an equidistant partition that divides the interval $[1, 2]$ into five subintervals. Use a calculator and record 6 significant decimal places. Compare your results, also with the actual integral to see that you came up with reasonable numbers.

(1) 5. We are considering the curve described by the equation:

$$xy^3 + x^2y = 6.$$

Find $\dfrac{dy}{dx}$ in general and at the point $(2,1)$ on the curve. Find the equation of the tangent line $t(x)$ and the normal line $n(x)$ at the point $(2,1)$. Use approximations of differentials to find an approximate value for $y(2.2)$.

(1) 6. A light house is located on a small island 5 miles off shore. The mirror in the light house revolves at an angular speed of 4 revolutions per minute. You are at the water's edge, 2 miles from the closest point on the shore to the island. How fast does the light beam go past you, whenever it reaches you?

(1) 7. Find the largest area rectangle inscribed into an ellipse given by the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

(1)  8. Find the fastest way to get from one point on the shore line of Fullmoon lake to an opposite point. As the name suggests, the lake is perfectly round, its radius is one kilometer, you can run at a pace of 10 kilometers per hour, and you can swim at a pace of 4 kilometers per hour.

(1) 9. Find the proportions of a largest volume open box with a hexagonal base.

(1) 10. Find the proportions of the largest volume cone inscribed in a ball of radius $R$.

(1) 11. Discuss the graph of the function $f(x) = (x-1)^2(x^2-1)$, and find its absolute extrema on $[-3/2, 3/2]$. This means, find the intercepts and where the function is positive, resp., negative. Find the critical points, where the function is increasing and decreasing, and its local extrema. Find where the function is concave up and down, and its inflection points. Find the absolute extrema, and sketch the function.

(1) 12. Repeat the previous problem with the function $f(x) = x/(x^2 + 1)$ on the interval $[-1, 2]$.

(1) 13. Show that the function $p(x) = x^3 + x^2 - x - 4$ has a zero between $x = 0$ and $x = 2$. A rigorous and complete argument is expected. Use Newton's method to find the zero, approximately. Make a table with your first guess $x_0$ and two improvements $x_1$ and $x_2$. You may use your calculator. Include a column with $p(x_0)$, $p(x_1)$, and $p(x_2)$ in your table.

(1) 14. Consider the region $\Omega$ bounded by the graph of the function $f(x) = \sqrt{x}$, the $y$–axis, and the line $y = 1$. Find the solids of resolution if $\Omega$ is revolved about the

(a) $x$–axis.

(b) line $y = 1$.

(c) $y$–axis.

(d) line $x = -1$.