Obituary:

R. Alan Day

It was with sadness that we learned in August, 1989 that Alan Day had undergone surgery in July to eliminate jaundice caused by an obstruction of the pancreas. This was the beginning of his sixteen month struggle against pancreatic cancer, ending with his death on November 26, 1990. Throughout this period, Alan pursued his mathematical research, mostly from an office he established in the basement of his home. Many of Alan's friends and colleagues visited Thunder Bay during this time.

In September 1990, Alan even made a trip to Connecticut to give a series of lectures on the doubling construction and collaborate with J. B. Nation. During his illness, Alan was supported and encouraged by his wife, Lise Minville. Notable in their support of both Alan and Lise were Christian Herrmann and Jiri Sichler.

Richard Alan Day was born on October 9, 1941 in Sault Ste. Marie, Ontario. He grew up and attended primary and high school in North Bay, Ontario, a few hundred miles away. Supported by the Canadian Air Force, he went to McMaster University in Hamilton, Ontario, to take an undergraduate degree in mathematics from 1959 until 1963. Following graduation, he spent the years 1963 until 1967 as the Lead Navigator in the 415 (M.P.) Squadron at the rank of Captain, resigning his commission in 1967. Even though Alan's undergraduate record was not outstanding, back in the Air Force he became quickly bored and there began his love of mathematics. He was admitted to graduate school at McMaster on the basis of work he had done while in the Air Force, and completed his Masters degree after only nine months, graduating in May 1968. His thesis, 'On Modular Equational Classes', was published in the Canadian Mathematical Bulletin as 'A characterization of modularity for congruence lattices of algebras', in 1969. This caused an explosion of publications on Mal'cev type conditions and the terms that he proved equivalent to modularity are now called Day-terms. Alan continued at McMaster and completed his Ph.D. degree under the supervision of Günter Bruns, ending with the oral defense of his Ph.D. thesis on April 15, 1970.

Having completed his doctorate, Alan accepted a National Research Council post-doctoral fellowship at Vanderbilt University in Nashville, Tennessee for the academic year 1970–1971, at the University of Waterloo in Ontario during the summer of 1970, and at the Technische Hochschule, in Darmstadt, Germany for the summer of 1971. In the fall of 1971, he returned to Northern Ontario as an Assistant Professor at Lakehead University. He was promoted to Associate Professor in 1975, and to Full Professor in 1980. His sabbatical years were divided between T.H. Darmstadt and Vanderbilt University as visiting professor, in 1975–1976, and between T.H. Darmstadt and the University of Hawaii, for 1983–1984. He also spent the winter of 1987 as an invited professor at the University of Hawaii.

Alan participated in many administrative activities for mathematical journals and societies. He was a member of the editorial board of Algebra Universalis from 1982 until illness forced suspension of these duties. He was elected a member of the Board of Directors of the Canadian Mathematical Society from 1987 until 1989, and a member of the N.S.E.R.C. Mathematics Grant Selection Committee from 1987 onwards. Alan refereed for several journals, gave invited addresses at many conferences, and worked on the organizing committee for specialized conferences funded by N.S.E.R.C. A long-standing N.S.E.R.C. supported scientist, he was awarded the Lakehead University Distinguished Researcher Award in 1990.

As we've mentioned, Alan's masters thesis [3] gave a Mal'cev condition for congruence modularity, i.e., a necessary and sufficient condition stating the existence of terms satisfying certain equations in order that all the congruence lattices

of a variety of algebras be modular. Jónsson had proved an analogous result for the distributivity of congruence lattices. This often cited Mal'cev condition has proved extremely important in the field of general algebra. At first one would guess that this result would be a straightforward generalization of Jónsson's result and that any lattice equation would have a similar Mal'cev condition. But the proof of Day's results is more difficult than Jónsson's, and it is not clear how to generalize either. In fact, for many years it was an open question if there was any lattice equation which had a Mal'cev condition other than distributivity, modularity, and the two trivial equations.

J. B. Nation, influenced by Alan's work, proved that there is a lattice equation, lattice theoretically weaker than modularity, which nevertheless implies congruence modularity whenever all the algebras of a variety satisfy it. This began the study of congruence varieties. Alan was in turn fascinated by this subject and wrote several important papers [9, 13, 15] about it early in his career. His proofs involved a subtle manipulation of terms and, when asked by a colleague about how he found one particularly hard proof, he replied "Simple. I came to work in the morning, wrote the equations down, and tried to manipulate them. After two years of doing this every day, I found the proof." Reference [32] characterized lattice equations which imply congruence modularity and answered several other fundamental questions in this area.

Also during the first half of his career, he made several contributions to the theory of free lattices. In [4] he gave a simple solution to the word problem for free lattices. This paper introduced his famous doubling construction, which he continued to use and improve throughout his career. One of his most important results, the fact that free lattices are weakly atomic, made heavy use of his doubling [14]. This result implies that splitting lattices generate the variety of all lattices, a fact important in the study of congruence varieties. Alan also showed that if a semidistributive lattice was a lower bounded image of a free lattice, it was upper bounded as well [17]. This result plays a critical role in Nation's proof of Jónsson conjecture on finite sublattices of free lattices, and in Freese and Nation's polynomial time algorithm for finding the lower covers of an element of a free lattice.

During the second half of his career, Alan became interested in modular and Arguesian lattices. (Arguesian lattices are a subclass of modular lattices satisfying an equation based on Desargues law. They are particularly important as they include all lattices of permuting equivalence relations and the lattices associated with the classical algebraic systems.) A series of important papers, written primarily with Jónsson [38, 41, 42, 43, 44], show that minimal failures of the Arguesian law lead to certain lattice configurations which capture the duality and symmetry in failures of Desargues law in projective planes, giving us a much clearer picture of the nature of minimal non-Arguesian lattices. In a different direction, Day and Pickering [50] showed that the techniques von Neumann used in his coordinatization results could be extended to show that Arguesian lattices containing a 3-frame determine a ring, which can be used to coordinatize part of the lattice.

Day and Ježek [40], proved the important reuslt that the only nontrivial variety of lattices with the amalgamation property is the class of distributive lattices. Grätzer, Jónsson, and Lakser had shown that there are no nondistributive modular varieties with the amalgamation property, but several people worked on the general problem over a 10 year period before it was finally solved by Day and Ježek.

Although he worked mostly in lattice theory in the latter part of his career, he still maintained an interest in general algebra, publishing two papers on the commutator [35, 45]. Also towards the end of his career, he found a new interest in the application of lattice-theoretic techniques to the theory of relational databases [29, 30].

These are only some of the highlights of Alan's substantial contribution to mathematics. One particularly impressive aspect of his mathematical career that is apparent from his publication list is that he never slowed down.

Personally, Alan always had a great affection for Northern Ontario, good food (which he often cooked himself), beer, fine wine, and good humour. He was very proud of being Canadian.

Alan Day applied the high standards of his research to all aspects of his life. This made him an inspiring, though demanding, teacher and an untiring advocate of excellence in general. He was also inflexible and argumentative at times. But underneath a sometimes gruff exterior, was a warm and sensitive person who gave his all to the people and things in which he believed.

Publications

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