(1) In this question, $R$ is a commutative ring with identity. This question continues the idea of speaking of divisibility in terms of ideals. An ideal $P$ in $R$ is called prime if $P \neq R$ and whenever $ab \in P$, then $a \in P$ or $b \in P$.

(a) Suppose $R$ is an integral domain. Show that, for $a \neq 0$, the principal ideal $(a)$ is prime if and only if $a$ is a prime element.

(b) Show that the zero ideal $\{0\}$ is prime if and only if $R$ is an integral domain.

(c) Let $I$ be an ideal of $R$. Show that $I$ is prime if and only if $R/I$ is an integral domain.

(d) Show that every maximal ideal is prime. Can you give an example of a non-maximal prime ideal?