(1) Consider the symmetric group on 3 things, $S_3$.
(a) How many elements does it have?
(b) Label these as you wish and write out a multiplication table for these elements.

(2) (a) Try to figure out how many symmetries an equilateral triangle has.
(b) Label these symmetries as you wish and write out a multiplication table for them.

(3) The group of integers $(\mathbb{Z}, +)$ under addition is what we call an *abelian* group. This is because if $m, n \in \mathbb{Z}$, then $m + n = n + m$ (i.e. addition is “commutative”). This is not true for $GL_2(\mathbb{R})$, so we say this group is *non-abelian*.
(a) Find two matrices $A, B \in GL_2(\mathbb{R})$ such that $A \cdot B \neq B \cdot A$.
(b) Find two matrices $C, D \in GL_2(\mathbb{R})$ such that $C \cdot D = D \cdot C$.
(c) Is $S_3$ abelian?

(4) Consider the ring of integers modulo 5, denoted $\mathbb{Z}/5\mathbb{Z} = \{0, 1, 2, 3, 4\}$.
(a) Write out the multiplication table of $\mathbb{Z}/5\mathbb{Z}$ (feel free to also do the addition table).
(b) Is $\mathbb{Z}/5\mathbb{Z}$ a field? (What I’m actually asking is: “Does every non-zero element of $\mathbb{Z}/5\mathbb{Z}$ have an inverse?”).

(5) If $m$ is a perfect square, show that $m$ is of the form $3q$ or $3q + 1$.

(6) If $d|m$ and $d|n$, show that $d|(5m - 20n)$

(7) Say $r \in \mathbb{Z}$ is a root of the polynomial $x^3 + ax^2 + bx + c$ (where $a, b, c \in \mathbb{Z}$). Show that $r|c$.

(8) If $2 \nmid a$, show that $4|a^2 - 1$. 