Assignment 8 – Part 1

(1) Recall from assignment 6 question (4) that, for $h_1, h_2 \in G$, we say that $h_2$ is conjugate to $h_1$ if there is $g \in G$ such that

$$h_2 = g^{-1}h_1g.$$

(The map from $G$ to itself sending $h$ to $g^{-1}hg$ is called “conjugation by $g$”).

(a) Show that “conjugacy” is an equivalence relation.

(b) For $h \in G$, we use the term “conjugacy class of $h$” to refer to the equivalence class under the above equivalence relation. We denote the conjugacy class of $h$ by $C_h$. Show that $\#C_h = 1$ if and only if $h \in Z(G)$.

(c) In question 4(b) of assignment 6, we found that for $D_n$ when $n$ is odd, the reflections are all in the same conjugacy class $C_r = \{\rho^i r : 0 \leq i < n\}$. Show that, for $n$ odd, all the other non-identity conjugacy classes in $D_n$ have size 2.

(d) When $n$ is even, the answer is slightly different. What are the conjugacy classes of $D_n$ in this case?

(2) Determine the conjugacy classes of the quaternion group $Q$. 