

## Solutions for Fair Division Homework 1

Dustin and Kendra want to split a bag of fun-sized candy, and decide to use the divider-chooser method. The bag contains 100 Snickers, 100 Milky Ways, and 100 Reese's, which Dustin values at \$1, \$5, and \$2 respectively. (This means Dustin values the 100 Snickers together at \$1, or \$0.01 for 1 Snickers).

1. If Kendra is the divider, and in one half puts: 25 Snickers, 20 Milky Ways, and 60 Reese's, what is the value of this half in Dustin's eyes? A. under \$2.00 **B. between \$2 and \$3** C. between \$3 and \$6 D. over \$6

$$\text{Dustin's value} = \frac{25}{100} \cdot \$1 + \frac{20}{100} \cdot \$5 + \frac{60}{100} \cdot \$2 = .25 + 1 + 1.20 = \$2.45$$

2. Does Dustin consider this a fair share? A. Yes **B. No**

Dustin thinks the whole haul is worth  $\$1 + \$5 + \$2 = \$8$ , and  $\$8/2 = \$4 > \$2.45$ , so No

3. If Dustin was a divider, which of these are possible divisions consistent with his value system.
- A. 50 Snickers, 50 Milky Ways, and 50 Reese's
  - B. 100 Snickers, 20 Milky Ways, and 100 Reese's
  - C. 200 Snickers, 0 Milky Ways, and 100 Reese's
  - D. 2 of A, B, and C**
  - E. All of A, B, and C

(A) is obviously half the value (since half of each candy)

(B) is worth  $\$1 + \frac{20}{100} \cdot \$5 + \$2 = \$(1+1+2) = \$4$ , which is  $\$8/2$

(C) is not a possible division! (There aren't 200 Snickers!)

So the answer is D since A & B work, C doesn't

4. Three players (one divider, two choosers) are going to use Lone Divider to cut a cake. The divider cuts the cake into 3 pieces. The two choosers value the cake as follows:

	P1	P2	P3
Chooser A	30%	40%	30%
Chooser B	32%	32%	36%

Which of the following is a fair division of the cake?

- A. Chooser A gets P3; chooser B gets P2; Divider gets P1
- B. Chooser A gets P1; chooser B gets P2; Divider gets P3
- C. Chooser A gets P2; chooser B gets P1; Divider gets P3
- D. Chooser A gets P2; chooser B gets P3; Divider gets P1**
- E. None of these

The only piece 'acceptable' to Chooser A is P<sub>2</sub> (since  $.4 > 1/3$ )

Similarly the only piece acceptable to Chooser B is P<sub>3</sub>.

Therefore the one and only fair division is B gets  $P_3$ , A gets  $P_2$ ,  
so the divider gets  $P_1$ .

Maggie, Meredith, Holly, and Zoe are dividing a piece of land using the lone-divider method. The values of the four pieces of land in the eyes of the each player are shown below.

	Piece 1	Piece 2	Piece 3	Piece 4
Maggie	21%	27%	32%	20%
Meredith	27%	29%	22%	22%
Holly	23%	14%	41%	22%
Zoe	25%	25%	25%	25%

5. Who was the divider?

- A. Maggie B. Meredith C. Holly **D. Zoe**

The divider always thinks all pieces are  $\approx$ . Since there are 4 people,  
 $\approx$  means 25%. Therefore, Zoe is the divider.

6. Which pieces do Meredith find acceptably large? A. {Piece 1} **B. {Piece 1, Piece 2}** C. {Piece 1,  
, Piece 2, Piece 3} D. {Piece 2, Piece 3} E. None of these

25% is the cutoff, so Meredith finds pieces 1 and 2 acceptable

7. Who gets Piece 1 in the final division?

- A. Maggie **B. Meredith** C. Holly D. Zoe

8. Who gets Piece 3 in the final division?

- A. Maggie B. Meredith **C. Holly** D. Zoe

Let's circle the acceptable pieces for the choosers (everyone but Zoe):

	Piece 1	Piece 2	Piece 3	Piece 4
Maggie	21%	<b>27%</b>	<b>32%</b>	20%
Meredith	<b>27%</b>	<b>29%</b>	22%	22%
Holly	23%	14%	<b>41%</b>	22%
Zoe	25%	25%	25%	25%

Holly must get Piece 3. Now Maggie has only 1 choice, Piece 2.  
That leaves Piece 1 for Meredith.