

Solutions for Fair Division Homework 2

A huge collection of low-value Magic cards appraised at \$100 is being divided by 5 kids (P_1, P_2, P_3, P_4, P_5) using the last-diminisher method. The players play in a fixed order, with P_1 first, P_2 second, and so on. In round 1, P_1 makes the first selection and makes a claim on a pile of cards. For each of the remaining players, the value of the *current* pile of cards at the time it is their turn is given in the following table:

	P_2	P_3	P_4	P_5
Value of the current pile of cards	\$15	\$22	\$18	\$19

1. Which player gets his or her share at the end of round 1?

- A. P_1 B. P_2 C. P_3 D. P_4 E. P_5

Note that Appraised value / # of players = \$20, so that's everyone's target for "fairness".

Player 1's original cut would have valued the pile at \$20.

Player 2 thought the pile was worth only \$15, so passes.

Player 3 thinks the pile is worth \$22, so trims it down to \$20.

Players 4 and 5 already had seen the pile as worth < \$20, now after trimming see it as worth even less, so they both pass.

Since player 3 was the last player to trim the pile, s/he gets it.

2. What is the value of the share to the player receiving it?

- A. \$20 B. \$15 C. \$22 D. \$18 E. \$19

As mentioned, Player 3 trimmed the pile to what s/he thought was a value of \$20.

3. 7 players are dividing a cake using the Fink protocol (also known as Method of Lone Chooser⁹). After the first six rounds the first 6 players each have what they believe to be $1/6$ of the cake. At this next (seventh) round, a bunch of cuts are going to be made. How many in total?

- A. 6 B. 36 C. 42 D. 49

Each of the 6 players cuts their slice into 7 pieces, so

42 pieces altogether.

Three players, A, B, and C, want to divide the objects in the figure below using the method of markers. Their markers have already been placed.



4. Who gets the first group of objects? A. A B. B C. C

5. How many pieces does B get (before leftovers are divided up)? A. 18 B. 8 C. 11 D. 12

6. How many leftover pieces are there? A. 0 B. 1 C. 2 D. 3 E. 4

One way to start is with a table:

	1 st group	2 nd group	3 rd group
A	1-9	10-18	19-30
B	1-10	11-18	19-30

C 1-11 12-20 21-30

The first "1st group" is A's (ends soonest), so A gets 1-9.

The first "2nd group" is B's (ends at 18 which is less than C's 20, we don't consider A any more since he has his share)
so B gets 11-18 (8 pieces)

Finally, C gets his 3rd group, which is 21-30.

Left over are 10, 19, and 20 (3 pieces)

As part of an inheritance, four children, Abby, Ben, Carla, and Dan, are dividing four vehicles using Sealed Bids. Their bids (in thousands of dollars) for each item is shown below. Use the Sealed Bid method to allocate the goods; answer the given questions about the final allocation.

	Abby	Ben	Carla	Dan
Motorcycle	6	7	10	8
Car	8	13	10	12
Tractor	3	2	5	4
Boat	7	6	3	8

7. Who inherits the car? A. Abby B. Ben C. Carla D. Dan

8. What is the total cash value of Ben's inheritance (in thousands)?

A. 8 B. 9 C. 10 D. 11 E. None of the above

9. Who inherits the most (total cash value)? A. Abby B. Ben C. Carla D. Dan

High bid for the motorcycle was Carla
" " " " car Ben (so he inherits it)
" " " " tractor " Carla
" " " " boat " Dan

Let's indicate this on the table:

	Abby	Ben	Carla	Dan
Motorcycle	6	7	10	8
Car	8	13	10	12
Tractor	3	2	5	4
Boat	7	6	3	8

$\frac{24}{4} = 6$ $\frac{28}{4} = 7$ $\frac{28}{4} = 7$ $\frac{32}{4} = 8$ ← Fair Market Share
 (= sum of bids / # of items)
 $0 - 6$ $13 - 7$ $15 - 7$ $8 - 8$
 $= -6$ $= 6$ $= 8$ $= 0$ ← Amount owed to estate

Estate now has $-6 + 6 + 8 + 0 = 8$ Thousand dollars,
divided 4 ways = 2 Thousand

So
Abby inherits no goods + 6 (The -6 she owes the estate) + 2 (estate leftover) = 8

Ben inherits $13 - 6$ (amount owed to estate) + 2 = 9

(By the way, if you have trouble understanding the point of these calculations, think of it this way: Ben got a car worth 13, but the fair market share of the estate based on his own valuations is only 7, that's why he

owes the estate $13-7=6$, so he's left with $(13-(13-7))=7$
 + whatever his share of the leftovers is (2),)

Carla inherits $15-8+2=9$

Dan inherits $8-0+2=10$, The most

10. A cake is to be divided among Roy, Sam, and Trish by the Selfridge-Conway variant of Lone Divider.
 The three players value the pieces after the first cut as follows:

	P1	P2	P3
Roy	10%	55%	35%
Sam	33.33%	33.33%	33.33%
Trish	40%	30%	30%

Who divides the trimmings? (This is after the trimmed pieces are already allocated to the diners.)

A. Roy B. Sam C. Trish (Both are correct!)

- 1) Sam is the initial divider (not of the trimmings) since the initial divider cuts the cake into equal pieces by his reckoning.
- 2) The 1st chooser, Roy, trims P2 down to 35%, puts the rest away.
- 3) Trish now chooses P1, obviously.
- 4) Roy's now up, he takes trimmed piece P2
- 5) Sam gets P3
- 6) Now Trish is the chooser who got the untrimmed piece, so Trish divides the trimmings.

(NOTE After Sam's initial division we could have decided that Trish was "Chooser A" in the algorithm and Roy was "chooser B". In that case, Trish would have trimmed P1 down to 30%, Roy would have chosen P2, Trish P1, and now Roy does the dividing of the trimmings. In other words, this exercise has 2 correct answers!)