Extra (review) problems for Exam 4 - Math 100, Spring 2015

Problem 1:

An island is to be divided among seven players ($P_1, P_2, P_3, \ldots, P_7$) using the last-diminisher method. The players play in a fixed order, with P_1 first, P_2 second, etc. P_3 gets his fair share at the end of round 1, and P_7 gets her fair share at the end of round 3. There are no diminishers in rounds 2, 4, and 5.

(a) Who is the last diminisher in round 1?

(b) Which player gets a fair share at the end of round 2?

(c) Which player cuts at the beginning of round 3?

(d) Which player gets a fair share at the end of round 4?

(e) Which player gets a fair share at the end of round 5?

(f) Which player is the chooser in the final round?

Problem 2:

An island is to be divided among eight players ($P_1, P_2, P_3, \ldots, P_8$) using the last-diminisher method. The players play in a fixed order, with P_1 first, P_2 second, etc. In rounds 1 and 5, everyone who has an opportunity to diminish does so. P_5 gets her fair share at the end of round 2. There are no diminishers in rounds 3 and 4. P_4 gets his fair share at the end of round 6.

(a) Which player gets a fair share at the end of round 1?

(b) Which player gets a fair share at the end of round 4?

(c) Which player gets a fair share at the end of round 5?

(d) How many diminishers are there in round 6?

(e) Which player is the divider in the final round?

Problem 3:

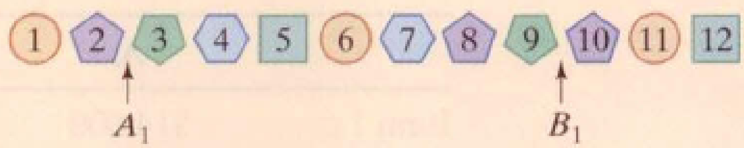
Three sisters (Ana, Belle, and Chloe) wish to use the method of sealed bids to divide up 4 pieces of furniture they shared as children. Their bids on each of the items are given in the following table.

	Ana	Belle	Chloe
Dresser	\$150	\$300	\$275
Desk	180	150	165
Vanity	170	200	260
Tapestry	400	250	500

Describe the final outcome of this fair-division problem.

Problem 4:

Two players (A and B) agree to divide the 12 items shown by lining them up in order and using the method of markers. The players' bids are as indicated.

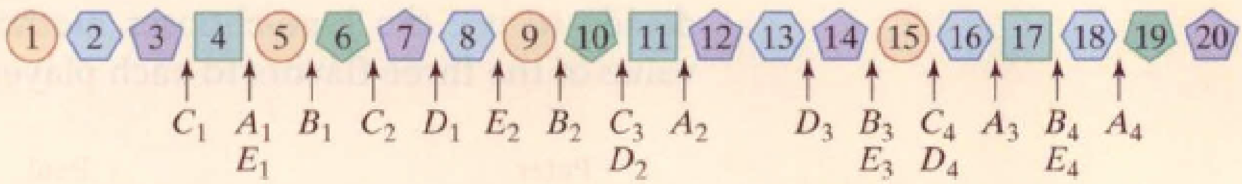


(a) Describe the allocation of items to each player.

(b) Which items are left over?

Problem 5:

Five players (A, B, C, D, and E) agree to divide the 20 items shown by lining them up in order and using the method of markers. The players' bids are as indicated.



(a) Describe the allocation of items to each player.

(b) Which items are left over?

Problem 6:

Preliminaries- There are 4 players: Brad, Stan, Lisa, and Allison. One of the players is randomly selected to be the divider by rolling a dice. Stan gets the lowest #, so he is the divider. Stan divides the cake into 4 shares, S1, S2, S3, and S4. Table 1 shows how Each of the players values the 4 shares.

Table 1

	S1	**S2**	**S3**	**S4**
Brad	50%	10%	10%	30%
Stan	25%	25%	25%	25%
Allison	40%	15%	15%	30%
Lisa	25%	25%	40%	10%

A. Indicate which of the 4 slices are fair shares to Brad.
B. Indicate which of the 4 slices are fair shares to Stan.
C. Indicate which of the 4 slices are fair shares to Allison.
D. Indicate which of the 4 slices are fair shares to Lisa.
E. Using S1, S2, S3, S4 as the four shares, describe a fair division of the cake.

The remainder of the problems are non-computational, and relate to the American Scientist article,

> "Mathematical Devices for Getting a Fair Share" by Theodore P. Hill, *American Scientist*, July/August 2000, pages 325-331

which I linked on the web page on April 6 (look for link labeled "Here is a very readible survey article from American Scientist (corrected)").

7. In the oldest known fair-division problem, why don't they just go for a proportional division?
8. What is the Ham Sandwich Theorem? Does it work for three shapes in two dimensions?
9. The article refers to a "round-table" procedure for dividing a cake. We discussed this procedure under a different name; which name?
10. What is a *super-fair* division?